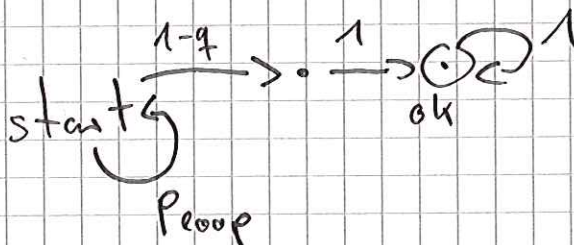


Series 2 - Solution

MUPS 14

① $P_x(\text{start} \models \Diamond \text{ok})$
a)

Observation: probability of all paths finally reaching ok-state is composed of probability to "loop on start-state" and of probability to go directly from start to ok without any loops.



$$P_{\text{ok}} = \cancel{\text{start}} (1-q) + p_{\text{loop}} \cdot (1-q) + p_{\text{loop}}^2 \cdot (1-q) \dots$$

$$\text{g.s.} = (1-q) \cdot \frac{1}{1-p_{\text{loop}}}$$

$$\begin{aligned} p_{\text{loop}} &= q \cdot (p^{n-1} \cdot (1-p) + p^{n-2} \cdot (1-p) + \dots + p^0 \cdot (1-p)) \\ &= q \cdot (1-p) \cdot \sum_{i=0}^{n-1} p^i \\ &= q \cdot \cancel{(1-p)} \cdot \frac{1-p^n}{1-p} = \boxed{q - qp^n} \end{aligned}$$

"after n times p , error state is reached"

1 b) $u=10, p=0.1, q=0.5$

$$P_{ok} = (1-0.5) \cdot \frac{1}{1 - (0.5 - 0.5 \cdot (0.1)^{10})}$$

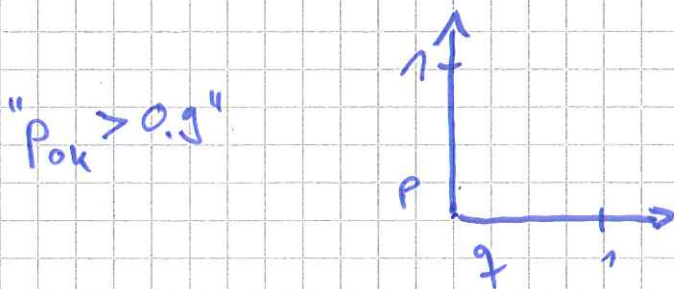
$$= \frac{0.5}{1 - 0.5 + 0.5 \cdot 10^{-10}} = \frac{0.5}{0.5 \cdot (1 + 10^{-10})}$$

$$\boxed{= 0.9}$$

c)

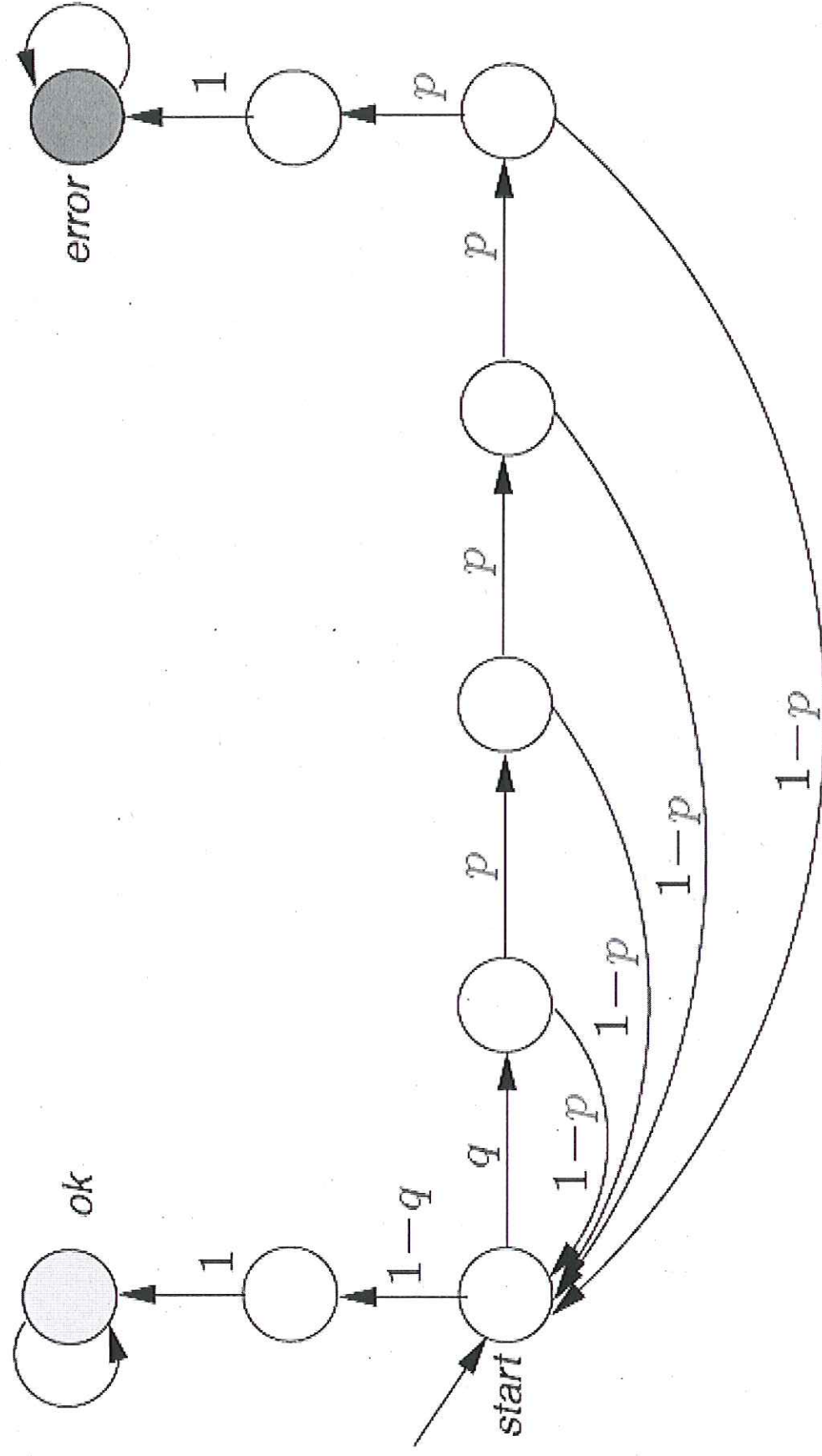
- such "parametric" formulas can be used to leave probabilities open

- using a method to give a solution space for functions like these, developers could estimate the impact of the different parameters:



- for instantiations of the parameters, a well-defined DTMC has to result

→ additional constraints



p = probability of message loss; q = probability of selecting occupied address

②

To show: "Asymptotic Behavior" is the same for connected states $s, s' \in S$.

Proof: s' is reachable from $s \Rightarrow \exists M \geq 0. p_{s,s'}(M) > 0$

$s \rightsquigarrow s'$
M steps

s is reachable from $s' \Rightarrow \exists N \geq 0. p_{s',s}(N) > 0$

$s' \rightsquigarrow s$
N steps

Chapman - Kolmogorov:

$$p_{s,s'}(u+v) \geq p_{s,s''}(u) \cdot p_{s'',s'}(v), \text{ for } s'' \in S$$

$$\underline{N+M \leq u} \quad (s'' = s')$$

$$p_{s',s'}(u) \geq p_{s',s}(N) \cdot p_{s,s'}(u-N)$$

$$\geq p_{s',s}(N) \cdot p_{s,s}(u-M-N) \cdot p_{s,s'}(M)$$

$s' \xrightarrow{M} s \xrightarrow{u-M-N} s' \xrightarrow{N} s'$
 $s \xrightarrow{N} s' \xrightarrow{u-M-N} s' \xrightarrow{M} s$

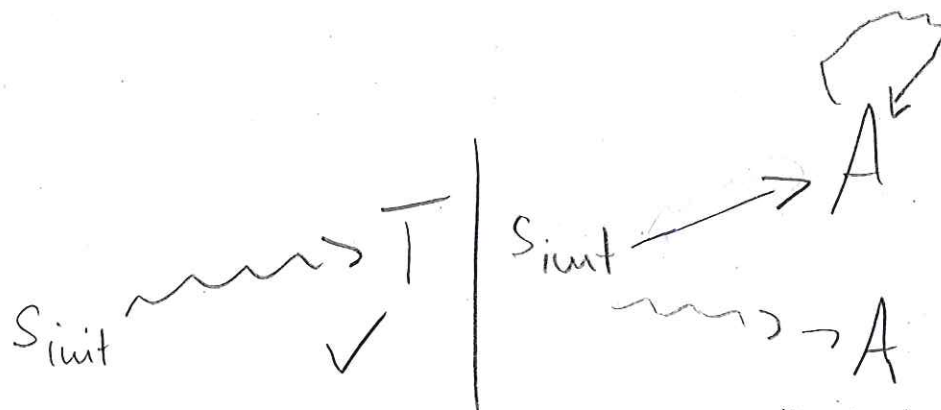
$$p_{s,s}(u) \geq p_{s,s'}(M) \cdot p_{s',s'}(u-M-N) \cdot p_{s',s}(N)$$

do not depend on u ! constants for fixed n !

There exist constants $A > 0, B > 0$ and (integers) R :

$$p_{s',s'}(u) \geq A \cdot p_{s,s}(u-R) \wedge p_{s,s}(u) \geq B \cdot p_{s',s'}(u-R)$$

$\Rightarrow$ same asymptotic behavior!



Exercise 3

(3 points)

- a) Recall that $Reach(s_{init}, T)$, the set of paths starting in s_{init} that reach a state in T , is measurable since it comprises the union of the cylinder sets of all finite paths ending in T .

The set of paths remaining forever in A is given by:

$$Path(s_{init}) \setminus Reach(s_{init}, S \setminus A)$$

This is measurable since $Reach(s_{init}, S \setminus A)$ is measurable, from the above, and the σ -algebra $\Sigma_{Path(s_{init})}$ is closed under complementation.

- b) The required set is the intersection of the set of paths from part (a) and:

$$\{s_0 s_1 s_2 \dots \in Path(s_{init}) \mid s_5 \in B\}.$$

The latter is the union of the cylinder sets of all finite paths $s_0 s_1 \dots s_5$ with $s_0 = s_{init}$ and $s_5 \in B$, and so is measurable. The σ -algebra is closed under intersection, yielding the result.

