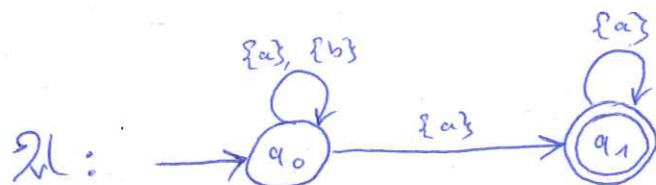


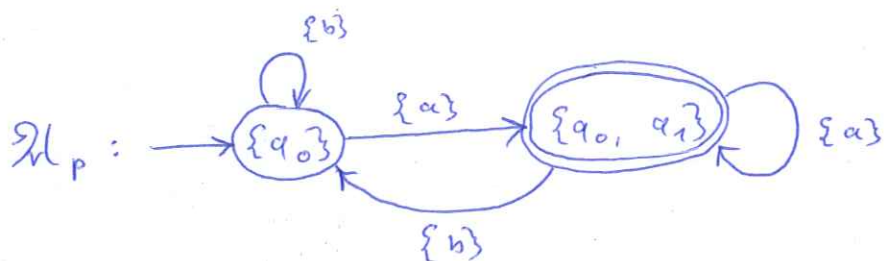
Series 3 - Solution

Exercise 1

a) NFA for $(a+b)^* a^\omega$:



Power set construction of $\mathcal{M}$:



Problem:

$(ab)^\omega \notin \mathcal{L}(\mathcal{M})$, but

$(ab)^\omega \in \mathcal{L}(\mathcal{M}_p)$ ⚡

b) Let $\mathcal{D} = (S, P, c_{init}, AP, L)$ be a DTMC and let $\mathcal{A} = (Q, \mathcal{P}(AP), \Delta, q_0, F)$ be an NBA.

Then $\mathcal{D} \boxtimes \mathcal{A} := (S \times Q, P', c'_{init}, \{accept\}, L')$, where:

$$P'((s, q), (s', q')) = \begin{cases} P(s, s'), & \text{if } (q, L(s')q') \in \Delta \\ 0, & \text{else} \end{cases}$$

$$c'_{init}((s, q)) = \begin{cases} c_{init}(s), & \text{if } (q_0, L(s), q) \in \Delta \\ 0, & \text{else} \end{cases}$$

$$L'((s, q)) = \begin{cases} \{accept\}, & \text{if } q \in F \\ \emptyset, & \text{else} \end{cases}$$

□

Now consider $\mathcal{D} \boxtimes \mathcal{A}$:

Both $c'_{init}((s_0, q_0)) = 1$ and $c'_{init}((s_0, q_1)) = 1$ $\Downarrow$

Exercise 2

a) True. Consider:

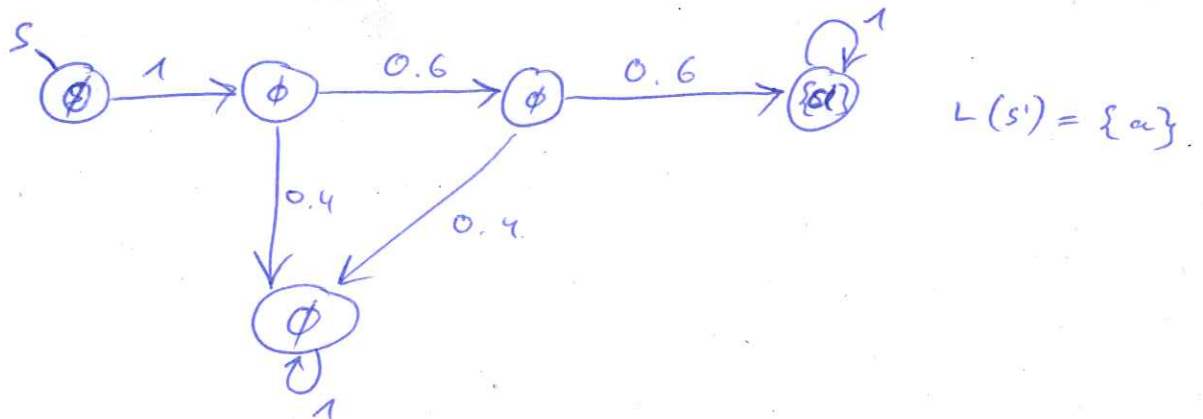
$$S \models IP_{=1}(\bigcirc IP_{=1}(\Box a))$$

$\Leftrightarrow$ "Pick ^{at random} any path $\sigma = s \cdot s_1 \cdot s_2 \dots$. Then almost ~~certainly~~ surely σ is of the form such that: $\sigma' = s_1 \cdot s_2 \dots$ visits a -states ad infinitum almost certainly"

$\Leftrightarrow$ "Pick at random any path $\sigma = s \cdot s_1 \cdot s_2 \dots$. Then almost surely the successor of every state along σ is almost surely an a -state"

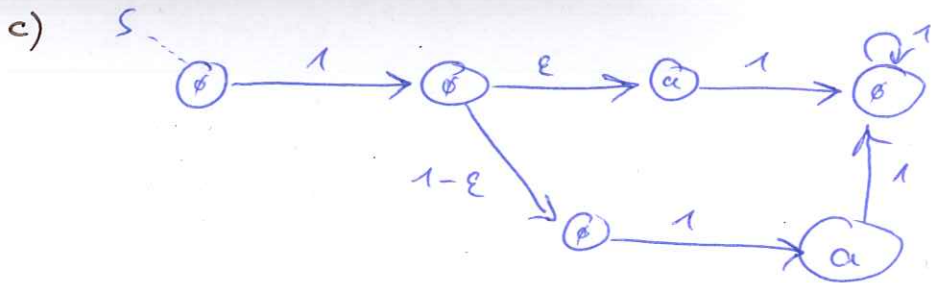
$$\Leftrightarrow S \models IP_{=1}(\Box IP_{=1}(\bigcirc a))$$

b) False. Counter example



$$S \not\models IP_{>0.5}(\bigcirc IP_{>0.5}(\Diamond a)) \quad , \quad \text{but}$$

$$S \models IP_{>0.5}(\Diamond IP_{>0.5}(\bigcirc a))$$



$$S \models \mathbb{P}_{=1}(\bigcirc \mathbb{P}_{=1}(\Diamond a)), \text{ but}$$

$$S \not\models \mathbb{P}_{=1}(\Diamond \mathbb{P}_{=1}(\bigcirc a))$$

Exercise 3

Algorithm for S_{no} :

1. Compute $R := \text{Sat}(\text{IP}_{>0}(\Phi \cup \Psi))$:

$R := \text{Sat}(\neg \Psi)$

done := false

while (done = false)

$R' := R \cup \{s \in \text{Sat}(\Phi) \mid \exists s' \in R : P(s, s') > 0\}$

if ($R' = R$)

done := true

end

$R := R'$

end

2. Return $S \setminus R = \text{IP}_{=0}(\Phi \cup \bar{\Psi})$

Algorithm for S_{yes} :

1. Compute $Sat(P_{<1}(\Phi \cup \Psi))$, i.e. the set of states with a non-zero probability of reaching S_{no} , passing only through Φ -states:

$R := S_{no}$

done := false

while (done = false)

$R' := R \cup \{s \in Sat(\Phi) \setminus Sat(\Psi) \mid$
 $\exists s' \in R : P(s, s') > 0\}$

if ($R = R'$)

done := true

end

$R := R'$

end

2. Return $S \setminus R$