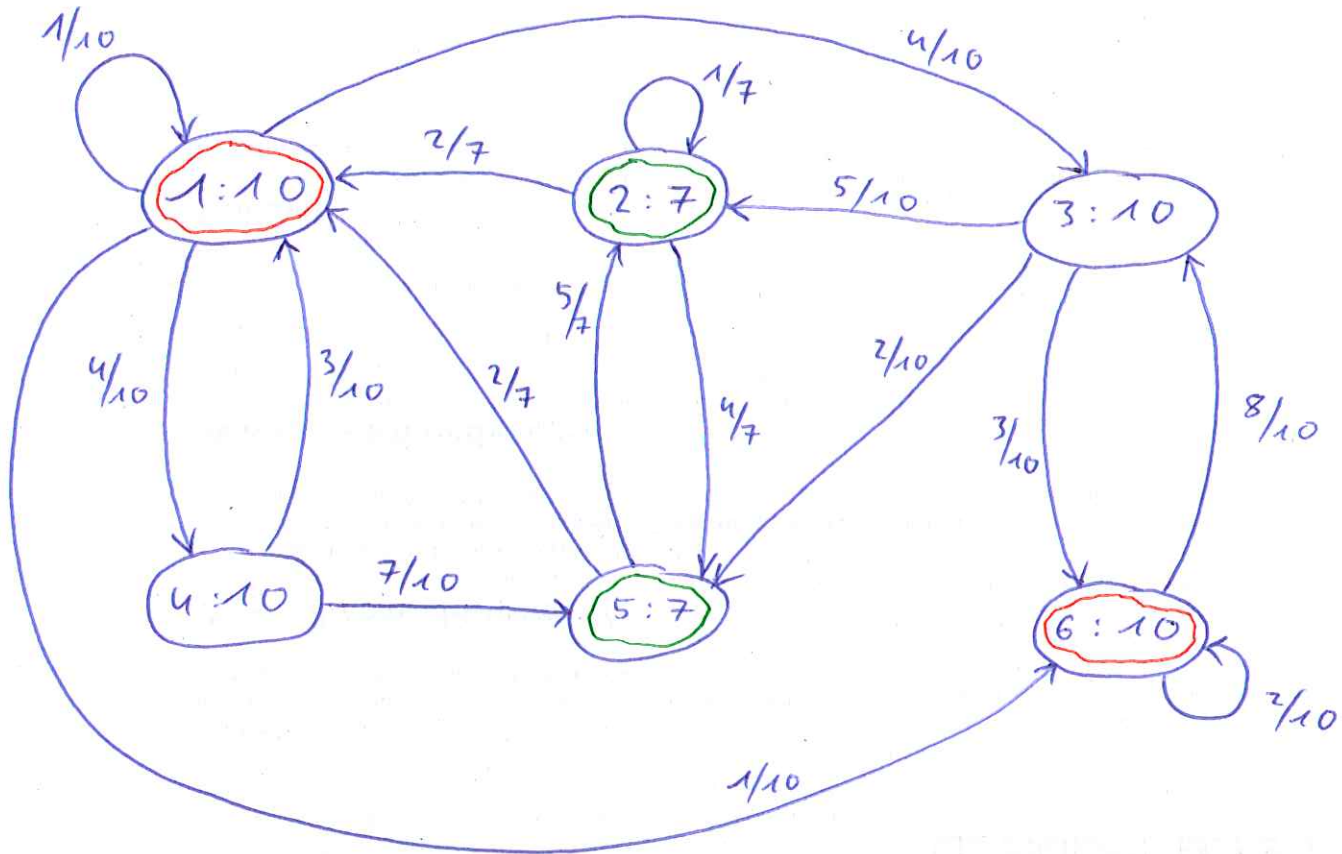


Series 8 - Solutions

Exercise 1

a) CTMC with rates and probabilities:



b)

$$\vec{P} \cdot \begin{bmatrix} 1 & 0 & 4 & 4 & 0 & 1 \\ 2 & 1 & 0 & 0 & 4 & 0 \\ 0 & 5 & 0 & 0 & 2 & 3 \\ 3 & 0 & 0 & 0 & 7 & 0 \\ 2 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 10 & & & & & \\ & 7 & & & & \\ & & 10 & & & \\ & & & 10 & & \\ & & & & 7 & \\ & & & & & 10 \end{bmatrix}$$

$$= \vec{P} \cdot \begin{bmatrix} -9 & 0 & 4 & 4 & 0 & 1 \\ 2 & -6 & 0 & 0 & 4 & 0 \\ 0 & 5 & -10 & 0 & 2 & 3 \\ 3 & 0 & 0 & -10 & 7 & 0 \\ 2 & 5 & 0 & 0 & -7 & 0 \\ 0 & 0 & 8 & 0 & 0 & -8 \end{bmatrix} \stackrel{!}{=} 0, \text{ s.t. } \|\vec{P}\|_1 = 1$$

$$\Rightarrow p_2 = \frac{323 p_1}{154}, \quad p_3 = \frac{5 p_1}{7}, \quad p_4 = \frac{2 p_1}{5}$$

$$p_5 = \frac{634 p_1}{385}, \quad p_6 = \frac{11 p_1}{28}$$

$$\|\vec{p}\|_1 = 1$$

$$\Rightarrow p_1 = \frac{140}{897}, \quad p_2 = \frac{3230}{9867}, \quad p_3 = \frac{100}{897},$$

$$p_4 = \frac{56}{897}, \quad p_5 = \frac{2776}{9867}, \quad p_6 = \frac{55}{897}$$

Exercise 2

Prob. mass. func of $\max\{X, Y\}$:

$$(\lambda_X \cdot e^{-\lambda_X \cdot x}) \cdot (\lambda_Y \cdot e^{-\lambda_Y \cdot x})$$

$$= \lambda_X \cdot \lambda_Y \cdot e^{-(\lambda_X + \lambda_Y) \cdot x} \quad \downarrow$$

Exercise 3

$$\frac{P(s, c)}{1 - P(s, [s])} = \frac{P(t, c)}{1 - P(t, [t])}$$

then

$$R(s, S' \setminus [s]) = P(s, S' \setminus [s]) \cdot r(s) = (1 - P(s, [s])) \cdot r(s)$$

$$\Leftrightarrow 1 - P(s, [s]) = \frac{R(s, S' \setminus [s])}{r(s)} \quad (*)$$

$$\stackrel{(*)}{\Rightarrow} \frac{P(s, c) \cdot r(s)}{R(s, S' \setminus [s])} = \frac{P(t, c) \cdot r(t)}{R(t, S' \setminus [t])}$$

$(*)(1.)$

$$\Leftrightarrow P(s, c) \cdot r(s) = P(t, c) \cdot r(s)$$

$$\Leftrightarrow R(s, c) = R(t, c)$$

$(*)$ Since $R(s, c) = R(t, c)$ for all $c \in S' \setminus \{[s]\}$, we obtain $R(s, S' \setminus [s]) = R(t, S' \setminus [t])$.

Divide only if $R(s, S' \setminus [s]) > 0$