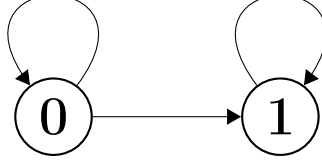


Introduction to Model Checking 2015:

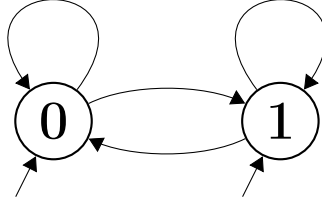
Exercise 1. Solution

Solution 1 1. A set E is countable if there exists a 1-1 mapping from set E to the set of natural numbers $\mathbb{N}$. Consider the transition system T_1 : The



set of executions is $\{0^\omega, 0^n 1^\omega \mid n \in \mathbb{N}\}$. The 1-1 mapping f is defined as follows: $f(0^\omega) = 0$ and $f(0^n 1^\omega) = n$.

2. Consider the transition system T_2 : The set of execution $E = (0+1)^\omega$. We



use Cantor's diagonalization argument to show that the set of execution of T_2 is not countable. Assume the contrary, that E is countable. Then there exists a 1-1 mapping f from E to $\mathbb{N}$. Arrange the elements of E as increasing sequence $\{w_i\}_{i \in \text{image}(f)}$ according to f . Now consider an execution w such that $w[i] \neq w_i[i]$ (where $w[i]$ is the i^{th} symbol of w). Clearly, w is different from every $w_i \in E$ yet $w \in (0+1)^\omega$. Contradiction.

Solution 2 1. • $S = S_1 \times S_2 \times S_3$, where $S_i = \{\text{on}_i, \text{off}_i\}$ for $i \in \{1, 2, 3\}$.

- $AP = S$, $Act = \{\text{flick}_i \mid i \in \{1, 2, 3\}\}$.
- $s_0 = (\text{off}_1, \text{off}_2, \text{off}_3)$.
- $L(s) = \{s\}$.
- $\langle (x_1, x_2, x_3), a, (x'_1, x'_2, x'_3) \rangle \in \rightarrow$ iff for exactly one $i \in \{1, 2, 3\}$ $x_i \neq x'_i$.

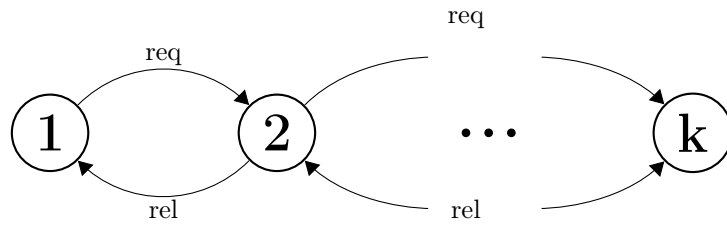
2. Number of states are 8.

3. The following execution is not a correct behaviour of the system:

$$(\text{off}_1, \text{off}_2, \text{off}_3) \xrightarrow{\text{flick}_1} (\text{off}_1, \text{on}_2, \text{off}_3) \xrightarrow{\text{flick}_2} (\text{off}_1, \text{off}_2, \text{off}_3)$$

Note: The anomaly in the above system behaviour is due to the fact that at each state (x_1, x_2, x_3) we forget the sequence of flicks used to arrive at it. If we store the mapping between the switch and the bulbs along with the state of the three light bulbs then we can describe a transition system where all execution are valid, albeit at a cost of complexity as now there are $8 \cdot 3!$ states.

Solution 3 1. The transition system of the arbiter is as follows:



2. The composition of three process and the arbiter is as follows:

