



Winter term 2013/14

Prof. Dr. Ir. J.-P. Katoen

**Exam in *Introduction to Model Checking* for
Bachelor of Science and Master of Science students
February 17, 2014**

Family name: _____

First name: _____

Student number: _____

Field of study: ☐ Informatik (B.Sc.)
☐ Informatik (M.Sc.)
☐ Software Systems Engineering (M.Sc.)
☐ Others: _____

Please note the following hints:

- Keep your student id card and a passport ready.
- The only allowed materials are
 - a copy of the book “Introduction to Model Checking”,
 - a copy of the lecture slides,
 - a dictionary.

No other materials (i.a. exercises, solutions, handwritten notes) are admitted.

- This test should have six pages (including this cover sheet).
- Write your name and student number on every sheet.
- Also use the back side of the pages if needed.
- Write with blue or black ink; do not use a pencil.
- Any attempt at deception leads to failure for this exam, even if it is detected only later.
- The editing time is **120 minutes**.

Question	Possible	Received
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	
Grade		

Question 1

(10 points)

Let P be an LT property. Further let $P_{\text{safe}} = \text{closure}(P)$ be a safety property. Prove:

- (a) Every bad prefix of P_{safe} is also a bad prefix of P .
- (b) Every bad prefix of P is also a bad prefix of P_{safe} .

Hint: one way to prove this, is to assume the contrary and deduce a contradiction.

Question 2

(10 points)

- (a) Prove or disprove that the following two LTL formula are equivalent: $\Diamond \bigcirc a \equiv \bigcirc \Diamond a$.
- (b) Show there is **no** equivalent LTL formula for the CTL formula $\forall \Diamond \forall \bigcirc a$.
- (c) Assume we know that $\forall \bigcirc \forall \Diamond a \equiv \bigcirc \Diamond a$ which means that for all transition systems TS :

$$TS \models_{CTL} \forall \bigcirc \forall \Diamond a \iff TS \models_{LTL} \bigcirc \Diamond a .$$

Based on this assumption and the results above argue whether the two CTL formulas $\forall \Diamond \forall \bigcirc a$ and $\forall \bigcirc \forall \Diamond a$ are equivalent.

Question 3

(10 points)

Let $\psi = (\Box a) \cup b$ and $AP = \{a, b\}$.

- (a) Transform ψ into an equivalent basic LTL-formula φ . The basic LTL syntax is given by the following context free grammar:

$$\varphi ::= \text{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \cup \varphi_2$$

- (b) Compute all elementary sets with respect to $\text{closure}(\varphi)$

Hint: There are 7 elementary sets.

- (c) Use the algorithm from the lecture to construct the GNBA $\mathcal{G}_\varphi$ with $\mathcal{L}_\omega(\mathcal{G}_\varphi) = \text{Words}(\varphi)$. Therefore

- define its set of initial states and its acceptance component,
- and for each elementary set B , define $\delta(B, B \cap AP)$.

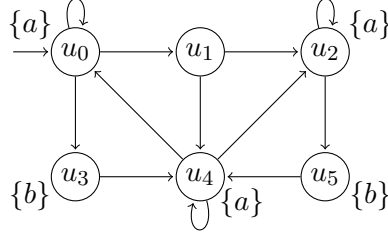
Question 4

(10 points)

Decide if $TS \models_{fair} \Phi$ for the CTL-formula Φ and the fairness assumption $fair$:

$$\Phi = \forall \square (a \rightarrow (\exists \bigcirc a \wedge \forall \diamond b))$$

$$fair = (\square \diamond (\neg a \wedge \neg b) \rightarrow \square \diamond b) \wedge (\diamond \square a \rightarrow \square \diamond b)$$

where TS over $AP = \{a, b\}$ is given by:

Proceed in the following steps:

- (a) Determine
- $Sat_{fair}(\exists \square \text{true})$
- .

Therefore give reachable strongly connected node sets that satisfy the fairness assumption.

- (b) Rewrite
- Φ
- into
- Ψ
- in existential normal form.

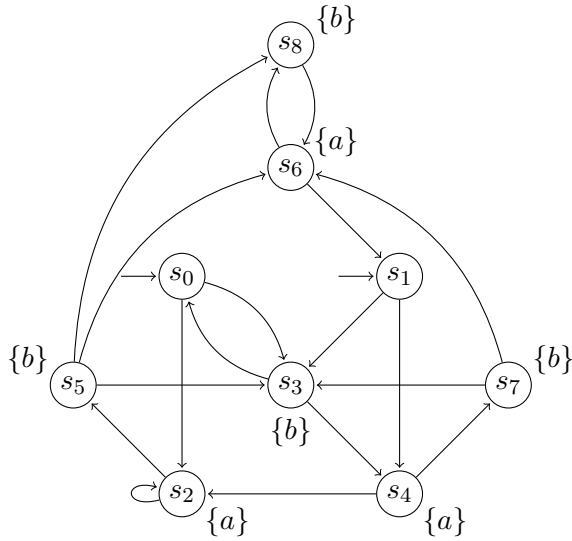
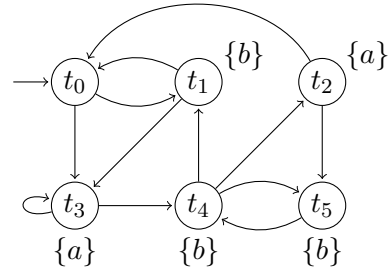
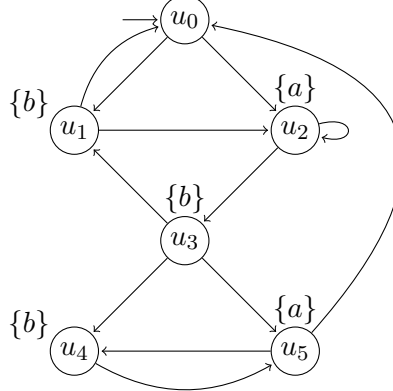
- (c) Determine
- $Sat_{fair}(\Psi)$
- .

Therefore determine $Sat_{fair}(\dots)$ for all subformulas starting with the smallest. You may give these sets directly but when determining $Sat_{fair}(\exists \square \Upsilon)$, give G_Υ , its SCCs and check fairness.

- (d) Decide
- $TS \models_{fair} \Phi$
- .

Question 5

(10 points)

Consider the three transition systems over $AP = \{a, b\}$: TS_1 : TS_2 : TS_3 :

- Decide whether $TS_1 \sim TS_2$. In case the systems are bisimilar provide a bisimulation relation, otherwise give a distinguishing CTL formula.
- Decide whether $TS_1 \sim TS_3$. In case the systems are bisimilar provide a bisimulation relation, otherwise give a distinguishing CTL formula.
- Based on your answers to questions a) and b), give an argument whether $TS_2 \sim TS_3$.