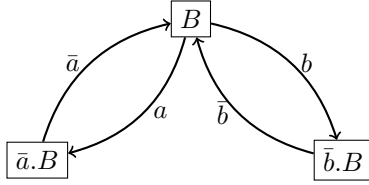


Concurrency Theory

Series 1 - Solution

Exercise 1 LTS and Derivations

LTS:



Derivations¹ (read from bottom to top):

$$\begin{array}{c}
 \text{(Act)} \frac{}{a.\bar{a}.B \xrightarrow{a} \bar{a}.B} \\
 \text{(Sum}_1\text{)} \frac{}{a.\bar{a}.B + b.\bar{b}.B \xrightarrow{a} \bar{a}.B} \\
 \text{(Call)} \frac{}{B \xrightarrow{a} \bar{a}.B}
 \end{array}$$

$$\begin{array}{c}
 \text{(Act)} \frac{}{b.\bar{b}.B \xrightarrow{b} \bar{b}.B} \\
 \text{(Sum}_2\text{)} \frac{}{a.\bar{a}.B + b.\bar{b}.B \xrightarrow{b} \bar{b}.B} \\
 \text{(Call)} \frac{}{B \xrightarrow{b} \bar{b}.B}
 \end{array}$$

$$\text{(Act)} \frac{}{\bar{b}.B \xrightarrow{b} B}$$

$$\text{(Act)} \frac{}{\bar{a}.B \xrightarrow{a} B}$$

Exercise 2 Sequential Composition and LTS

a) If it is possible to go from P to its direct successor P' ($P \xrightarrow{\alpha} P'$) use that way first ($P; Q \xrightarrow{\alpha} P'; Q$).

$$\text{(Seq}_1\text{)} \frac{P \xrightarrow{\alpha} P'}{P; Q \xrightarrow{\alpha} P'; Q}$$

Example: For the sequence $a.(\bar{a}.B)$ the successor is $\bar{a}$. Execute a first: $a.(\bar{a}.B) \xrightarrow{a} \bar{a}.B$. If there is no successor anymore ($P \not\xrightarrow{\alpha}$) go to the successor (Q') of the second element of the sequence (Q):

$$\text{(Seq}_2\text{)} \frac{P \not\xrightarrow{\alpha} \quad Q \xrightarrow{\alpha} Q'}{P; Q \xrightarrow{\alpha} Q'}$$

In the example the second element of the sequence is $\bar{a}.B$ and we move from there to its successor B .

b) *Isomorphism*: Two labelled transition systems are isomorphic if their structure is exactly the same. Isomorphic LTSs are only allowed to differ in their labelling of states. Choose an easy form for P and Q . Suppose that $P = Q = \text{nil}$.

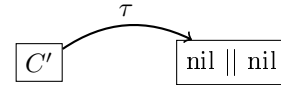
LTS(C'):

$$\boxed{C'}$$

LTS(C) is a little bit more complex. First, write down the derivations:

$$\begin{array}{c}
 \text{(Act)} \frac{}{\bar{a}.\text{nil} \xrightarrow{\bar{a}} \text{nil}} \quad \text{(Act)} \frac{}{a.\text{nil} \xrightarrow{a} \text{nil}} \\
 \text{(Com)} \frac{}{\bar{a}.\text{nil} \parallel a.\text{nil} \xrightarrow{\tau} \text{nil} \parallel \text{nil}} \\
 \text{(Res)} \frac{}{\bar{a}.\text{nil} \parallel a.\text{nil} \setminus \{a, \bar{a}\} \xrightarrow{\tau} \text{nil} \parallel \text{nil}} \\
 \text{(Call)} \frac{}{C \xrightarrow{\tau} \text{nil} \parallel \text{nil}}
 \end{array}$$

(Remember: τ is an unobservable action; Explanation to the derivation: For $P = Q = \text{nil}$: C is $(\bar{a}.\text{nil} \parallel \text{nil}[a.\text{nil} \setminus \text{nil}]) \setminus \{a, \bar{a}\}$. Replace nil with $a.\text{nil}$: $C = (\bar{a}.\text{nil} \parallel a.\text{nil}) \setminus \{a, \bar{a}\}$. In the next step the restriction is removed. Then the derivation is split for two parallel processes. Now one can see that in each process we get to nil . This result is added in a last line (Call) and on the right side of every line.) Now draw the LTS:



The LTSs are different and we conclude: $\text{LTS}(C') \not\cong \text{LTS}(C)$

Exercise 3

$$\begin{aligned}
 L \subseteq \Sigma^+ \text{ is regular} &\Leftrightarrow \text{ex. DFA } \mathcal{A}_L \text{ with } L(\mathcal{A}_L) = L \\
 &\Leftrightarrow \text{ex. NFA } \mathcal{A}_L' = (Q, \Sigma, q_0, \Delta, F) \text{ with } L(\mathcal{A}_L') = L \\
 &\text{and each } q \in Q \text{ is productive} \\
 & (= \text{it is possible to reach a final state from } q)
 \end{aligned}$$

Claim: There is a NFA $\mathcal{A}_L''$ with $\text{pref}(L) = L(\mathcal{A}_L'')$, where $\mathcal{A}_L'' = (Q, \Sigma, q_0, \Delta, Q)$. Let $w \in \Sigma^*$.

$$\begin{aligned}
 w \in L(\mathcal{A}_L'') &\Leftrightarrow \text{there exists a run of } \mathcal{A}_L'' \text{ on } w \text{ that ends in a final state of } \mathcal{A}_L'' \\
 &\Leftrightarrow \text{there exists a run of } \mathcal{A}_L'' \text{ on } w \text{ that ends in a productive state of } \mathcal{A}_L'' \\
 &\Leftrightarrow \text{ex. } w' \in \Sigma^* \text{ and } ww' \in L(\mathcal{A}_L') \\
 &\Leftrightarrow w \in \text{pref}(L)
 \end{aligned}$$

¹More examples: http://www.pst.ifi.lmu.de/Lehre/sose-2013/formale-spezifikation-und-verifikation/fotse_10.pdf