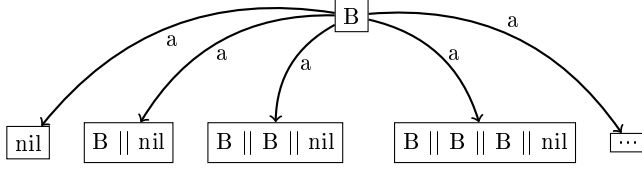


Concurrency Theory
Series 2 - Solution

Exercise 1 CCS and LTS's

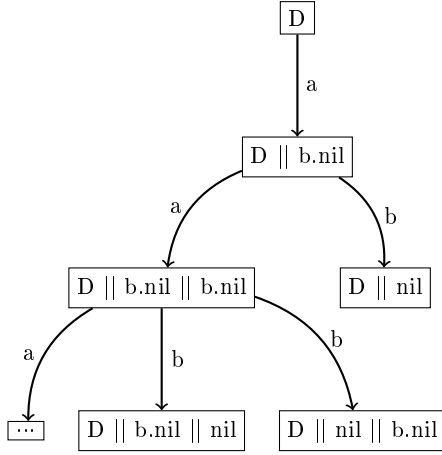
(a)

i) LTS:



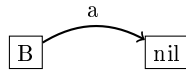
The LTS is infinite. $L(B) = a^*$, so $L(B)$ is regular.

ii) LTS:



The LTS is infinite. $L(D) = \{w \in \{a, b\}^* \mid \forall w' \preceq w : \#_a(w') \geq \#_b(w')\}$, so $L(D)$ is not regular.

iii) LTS:



The LTS is finite. $L(B) = \epsilon + a = \{\epsilon, a\}$, so $L(B)$ is regular.

b) No, for a counter example see a) iii).

Exercise 2 Complementation of HML Formulae a)

Assume $\llbracket F^C \rrbracket = S \setminus \llbracket F \rrbracket$.

To show: $\llbracket ([\alpha]F)^C \rrbracket = S \setminus \llbracket [\alpha]F \rrbracket$.

$$\begin{aligned}
 \llbracket ([\alpha]F)^C \rrbracket &= \llbracket \langle \alpha \rangle F^C \rrbracket = \langle \cdot \alpha \cdot \rrbracket \llbracket F^C \rrbracket \\
 &= \{s \in S \mid \exists s' \in \llbracket F^C \rrbracket : s \xrightarrow{\alpha} s'\} \\
 &\stackrel{*}{=} \{s \in S \mid \exists s' \in S \setminus \llbracket F \rrbracket : s \xrightarrow{\alpha} s'\} \\
 &= \{s \in S \mid \exists s' \in S : s' \notin \llbracket F \rrbracket \wedge s \xrightarrow{\alpha} s'\} \\
 &= S \setminus \{s \in S \mid \forall s' \in S : s' \in \llbracket F \rrbracket \vee s \not\xrightarrow{\alpha} s'\} \\
 &\stackrel{**}{=} S \setminus \{s \in S \mid \forall s' \in S : s \xrightarrow{\alpha} s' \Rightarrow s' \in \llbracket F \rrbracket\} \\
 &\stackrel{**}{=} S \setminus [\cdot \alpha \cdot](\llbracket F \rrbracket) = S \setminus \llbracket [\alpha]F \rrbracket
 \end{aligned}$$

$$**a \vee \neg b = \neg b \vee a = b \rightarrow a$$

c) This is true. Proof by induction on structure of F.

Induction Base:

$$F = \text{ff} : ((\text{ff})^C)^C = (\text{tt})^C = \text{ff}$$

$$F = \text{tt} : ((\text{tt})^C)^C = (\text{ff})^C = \text{tt}$$

$$\text{Inductive Hypothesis (I.H.): } (F_1^C)^C = F_1, (F_2^C)^C = F_2$$

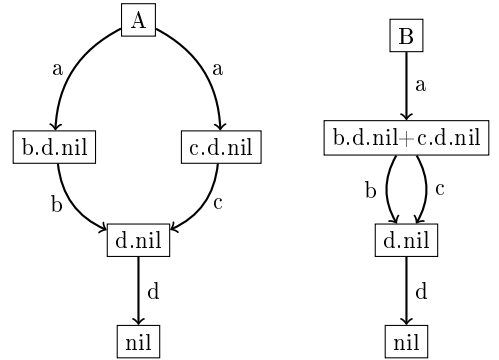
Inductive Step:

$$\begin{aligned}
 F = F_1 \wedge F_2 : ((F_1 \wedge F_2)^C)^C &= (F_1^C \vee F_2^C)^C \\
 &= (F_1^C)^C \wedge (F_2^C)^C \stackrel{\text{I.H.}}{=} F_1 \wedge F_2 \\
 F = F_1 \vee F_2 : ((F_1 \vee F_2)^C)^C &= (F_1^C \wedge F_2^C)^C \\
 &= (F_1^C)^C \vee (F_2^C)^C \stackrel{\text{I.H.}}{=} F_1 \vee F_2 \\
 F = \langle \alpha \rangle F_1 : ((\langle \alpha \rangle F_1)^C)^C &= ([\alpha]F_1^C)^C \\
 &= \langle \alpha \rangle (F_1^C)^C \stackrel{\text{I.H.}}{=} \langle \alpha \rangle F_1 \\
 F = [\alpha]F_1 : (([\alpha]F_1)^C)^C &= (\langle \alpha \rangle F_1^C)^C \\
 &= [\alpha](F_1^C)^C \stackrel{\text{I.H.}}{=} [\alpha]F_1
 \end{aligned}$$

□

b) see c)

Exercise 3 Construction of HML Formulae



a) $A \models \text{tt} \neq B$

b) $A \not\models [a]\langle b \rangle \text{tt} \neq B$

(All a -labelled transitions in B lead to a state where it is possible to get to tt with b).

c) $A \models \langle a \rangle [b] \text{ff} \neq B$

(It is possible for A to get in a state with a where all b -labelled transitions lead to a state ff. Note: $([a]\langle b \rangle \text{tt})^C = \langle a \rangle [b] \text{ff}$)