

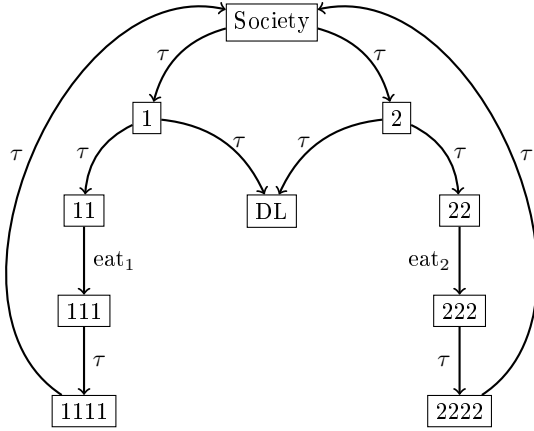
Concurrency Theory
Series 4 - Solution

Exercise 1 Dining Philosophers

a)

Society = (Phil₁ || Phil₂ || Spoon || Fork) \
 {pickUpFork, releaseFork, pickUpSpoon, releaseSpoon}
 Phil₁ = pickUpFork.pickUpSpoon.eat₁.releaseFork.
 releaseSpoon.Phil₁
 Phil₂ = pickUpSpoon.pickUpFork.eat₂.releaseSpoon.
 releaseFork.Phil₂
 Spoon = $\overline{\text{pickUpSpoon}}$. $\overline{\text{releaseSpoon}}$.Spoon
 Fork = $\overline{\text{pickUpFork}}$. $\overline{\text{releaseFork}}$.Fork

b)



1 = (pickUpSpoon.eat₁.releaseFork.releaseSpoon.Phil₁ || Phil₂ ||
 Spoon || $\overline{\text{releaseFork}}$.Fork) \ {...}

2 = (Phil₁ || pickUpFork.eat₂.releaseSpoon.releaseFork.Phil₂ ||
 $\overline{\text{releaseSpoon}}$.Spoon || Fork) \ {...}

DL = (pickUpSpoon.eat₁.releaseFork.releaseSpoon.Phil₁ ||
 pickUpFork.eat₂.releaseSpoon.releaseFork.Phil₂ ||
 $\overline{\text{releaseSpoon}}$.Spoon || $\overline{\text{releaseFork}}$.Fork) \ {...}

11 = (eat₁.releaseFork.releaseSpoon.Phil₁ || Phil₂ ||
 $\overline{\text{releaseSpoon}}$.Spoon || $\overline{\text{releaseFork}}$.Fork) \ {...}

111 = (releaseFork.releaseSpoon.Phil₁ || Phil₂ ||
 $\overline{\text{releaseSpoon}}$.Spoon || $\overline{\text{releaseFork}}$.Fork) \ {...}

1111 = (releaseSpoon.Phil₁ || Phil₂ || $\overline{\text{releaseSpoon}}$.Spoon || Fork) \ {...}

22 = (Phil₁ || eat₂.releaseSpoon.releaseFork.Phil₂ ||
 $\overline{\text{releaseSpoon}}$.Spoon || $\overline{\text{releaseFork}}$.Fork) \ {...}

222 = (Phil₁ || releaseSpoon.releaseFork.Phil₂ ||
 $\overline{\text{releaseSpoon}}$.Spoon || $\overline{\text{releaseFork}}$.Fork) \ {...}

2222 = (Phil₁ || releaseFork.Phil₂ || Spoon || $\overline{\text{releaseFork}}$.Fork) \ {...}

c) $D \stackrel{\text{max}}{=} \langle \text{Act} \rangle \text{tt} \wedge [\text{Act}] D$

d) Pick up fork first!

Exercise 2 Mutually Recursive Equation Systems

Iteration for X_2 :

$$0 : X_2 = \{s_0, s_1, s_2, s_3, s_4\}$$

$$1 : X_2 = \{s_0, s_2, s_3\}$$

$$2 : X_2 = \{s_0, s_2, s_3\}$$

Iteration for X_1 :

$$0 : X_1 = \emptyset$$

$$1 : X_1 = \{s_0, s_2, s_3\}$$

$$2 : X_1 = \{s_0, s_1, s_2, s_3\}$$

$$3 : X_1 = \{s_0, s_1, s_2, s_3\}$$

Exercise 3 HML with One Variable

Given:

$$\neg \exists F \in \text{HML} \forall \text{LTS} = (S, \text{Act}, \rightarrow) : |S| \geq 2 \\ \Rightarrow (\neg \llbracket F \rrbracket(S) = S \wedge \neg \llbracket F \rrbracket(\emptyset) = \emptyset)$$

which is the same as

$$\forall F \in \text{HML} \exists \text{LTS} = (S, \text{Act}, \rightarrow) : |S| \geq 2 \\ \wedge (\llbracket F \rrbracket(S) = S \vee \llbracket F \rrbracket(\emptyset) = \emptyset)$$

Choose $\text{LTS} = (\{s_0, s_1\}, \emptyset, \emptyset)$. Proof by induction.

Induction Base:

$$\llbracket X \rrbracket(M) = M \Rightarrow \llbracket X \rrbracket(S) = S \wedge \llbracket X \rrbracket(\emptyset) = \emptyset$$

$$\llbracket \text{tt} \rrbracket(S) = S$$

$$\llbracket \text{ff} \rrbracket(\emptyset) = \emptyset$$

Inductive Hypothesis (I.H): The above assumption $(\llbracket F \rrbracket(S) = S \vee \llbracket F \rrbracket(\emptyset) = \emptyset)$ holds for all proper subformulas of F .

Inductive Step:

$F = F_1 \wedge F_2$:

Case 1 : $\llbracket F_1 \rrbracket(S) = S \wedge \llbracket F_2 \rrbracket(S) = S$

$$\Rightarrow \llbracket F_1 \wedge F_2 \rrbracket(S) \stackrel{\text{Def.}}{=} \llbracket F_1 \rrbracket(S) \cap \llbracket F_2 \rrbracket(S) \stackrel{I.H}{=} S$$

Case 2 : $\llbracket F_1 \rrbracket(S) = S \wedge \llbracket F_2 \rrbracket(\emptyset) = \emptyset$ (or flipped)

$$\Rightarrow \llbracket F_1 \wedge F_2 \rrbracket(\emptyset) \stackrel{\text{Def.}}{=} \llbracket F_1 \rrbracket(\emptyset) \cap \llbracket F_2 \rrbracket(\emptyset) \stackrel{I.H}{=} \emptyset$$

Case 3 : $\llbracket F_1 \rrbracket(\emptyset) = \emptyset \wedge \llbracket F_2 \rrbracket(\emptyset) = \emptyset$

$$\Rightarrow \llbracket F_1 \wedge F_2 \rrbracket(\emptyset) \stackrel{\text{Def.}}{=} \llbracket F_1 \rrbracket(\emptyset) \cap \llbracket F_2 \rrbracket(\emptyset) \stackrel{I.H}{=} \emptyset$$

$F = \langle a \rangle F_1$:

$$\llbracket \langle a \rangle F_1 \rrbracket(M) = \emptyset \text{ (for the given LTS)} \Rightarrow \llbracket \langle a \rangle F_1 \rrbracket(\emptyset) = \emptyset$$

$F = F_1 \vee F_2$ and $F = [a] F_1$ analogous