

Exercise 1 Value Passing Process Definitions

a)

$$\begin{aligned} \text{Counter} &= (\text{Adder} \parallel \text{Buffer} \parallel \overline{\text{ain}}.\text{nil}) \setminus \{\text{ain}, \text{aout}\} \\ \text{Adder} &= \text{ain}(x).\overline{\text{out}}(x).\text{Adder}'(x) \\ \text{Adder}'(x) &= \overline{\text{aout}}(x+1).\text{Adder} \\ \text{Buffer} &= \text{aout}(x).\overline{\text{ain}}(x).\text{Buffer} \end{aligned}$$

b)

$$\begin{aligned} \text{Squares} &= (\text{Counter} \parallel \text{Squarer}) \setminus \{\text{out}\} \\ \text{Squarer} &= \text{out}(x).\text{square}(2x^2).\text{Squarer} \end{aligned}$$

Exercise 2 Translation of Value Passing into Pure CCS

Induction Base:

$$\begin{aligned} \text{nil} &: \text{nil} \xrightarrow{a(z)} \widehat{\text{nil}} \\ \widehat{\text{nil}} &= \text{nil} \xrightarrow{a(z)} \widehat{\text{nil}} \\ a(x).P &: a(x).P \xrightarrow{a(z)} P[z \setminus x] \\ \widehat{a(x).P} &= \sum_{z \in \mathbb{Z}} a_z.P[z \setminus x] \xrightarrow{a_z} \widehat{P[z \setminus x]} \\ \overline{a}(e).P &: \overline{a}(e).P \xrightarrow{a(z)} P \\ \widehat{\overline{a}(e).P} &= \overline{a_z}.\widehat{P} \xrightarrow{\overline{a_z}} \widehat{P} \\ \tau.P &: \tau.P \xrightarrow{\tau} P \\ \widehat{\tau.P} &= \tau.\widehat{P} \xrightarrow{\tau} \widehat{P} \end{aligned}$$

Inductive Hypothesis (I.H): The assumption $(P \xrightarrow{a(z)} P' \Leftrightarrow \widehat{P} \xrightarrow{a_z} \widehat{P'})$ holds for all proper subformulas $(P_1$ and $P_2)$ of P .

Inductive Step:

$$\begin{aligned} P_1 + P_2 &: P_1 + P_2 \xrightarrow{a(z)} P'_1 \\ \widehat{P_1 + P_2} &= \widehat{P_1} + \widehat{P_2} \xrightarrow{a_z}_{I.H.} \widehat{P'_1} \\ P_1 + P_2 &\xrightarrow{a(z)} P'_2 \text{ analogous} \\ P_1 \parallel P_2 &: P_1 \parallel P_2 \xrightarrow{a(z)} P'_1 \parallel P_2 \\ \widehat{P_1 \parallel P_2} &= \widehat{P_1} \parallel \widehat{P_2} \xrightarrow{a_z}_{I.H.} \widehat{P'_1} \parallel \widehat{P_2} = \widehat{P'_1 \parallel P_2} \\ P_1 \parallel P_2 &\xrightarrow{a(z)} P_1 \parallel P'_2 \text{ analogous} \\ P_1 \setminus L &: P_1 \setminus L \xrightarrow{a(z)} P'_1 \setminus L \\ \widehat{P_1 \setminus L} &= \widehat{P_1} \setminus \{a_y \mid a \in L, y \in \mathbb{Z}\} \\ &\xrightarrow{a_z} \widehat{P'_1} \setminus \{a_y \mid a \in L, y \in \mathbb{Z}\} = \widehat{P'_1 \setminus L} \\ P_1[f] &: P_1[f] \xrightarrow{f(a)(z)} P'_1[f] \\ \widehat{P_1[f]} &= \widehat{P_1}[\widehat{f}] \xrightarrow{f(a)_z} \widehat{P'_1}[\widehat{f}] = \widehat{P'_1[f]} \end{aligned}$$

$$\text{if } b \text{ then } P_1 : (b \text{ true}) \text{ if } b \text{ then } P_1 \xrightarrow{a(z)} P'_1$$

$$(b \text{ true}) \widehat{\text{if } b \text{ then } P_1} = \widehat{P_1} \xrightarrow{a(z)} \widehat{P'_1}$$

$$(b \text{ false}) \text{ if } b \text{ then } P_1 \xrightarrow{a(z)} \text{nil}$$

$$(b \text{ false}) \widehat{\text{if } b \text{ then } P_1} = \text{nil} \xrightarrow{a(z)} \text{nil}$$

$$C(e_1, \dots, e_k) : C(e_1, \dots, e_k) \xrightarrow{a(z)} P'_1$$

$$\widehat{C(e_1, \dots, e_k)} = \widehat{P_1[z_1 \setminus y_1, \dots, z_k \setminus x_k]} \xrightarrow{a_z} \widehat{P'_1}$$

Exercise 3 Structural Congruence in the π -Calculus

a)

$$\begin{aligned} &(\text{new } x \ Q_1 \not\equiv Q_1 \Rightarrow x \in f_n(Q_1)) \\ &\Leftrightarrow (x \notin f_n(Q_1) \Rightarrow \text{new } x \ Q_1 \equiv Q_1) \end{aligned}$$

Show the second formula:

$$\begin{aligned} Q_1 &\stackrel{(3.)}{\equiv} Q_1 \parallel \text{nil} \stackrel{(4.)}{\equiv} Q_1 \parallel \text{new } x \ \text{nil} \stackrel{(*,4.)}{\equiv} \text{new } x (Q_1 \parallel \text{nil}) \\ &\stackrel{(3.)}{\equiv} \text{new } x \ Q_1 \quad (* = x \notin f_n(Q_1)) \end{aligned}$$

b)

1. Application of process constructs on both sides ✓
2. Renaming of bound variable names ✓
3. Commutation and reassociation ✓
4. $f_n(P \parallel \text{nil}) \ (P \parallel \text{nil} = P) : f_n(P \parallel \text{nil}) = f_n(P) \cup f_n(\text{nil}) = f_n(P) \quad \checkmark$
5. $f_n(\text{nil}) \ (\text{nil} = \text{new } x \ \text{nil}) : f_n(\text{nil}) = \emptyset = \emptyset \setminus \{x\} = f_n(\text{new } x \ \text{nil}) \quad \checkmark$
6. $f_n(\text{new } x \ y \ P) \ (\text{new } x \ y \ P = \text{new } y \ x \ P) : f_n(P) \setminus \{x, y\} = f_n(\text{new } y \ x \ P) \quad \checkmark$
7. Let $x \in f_n(P)(*)$. Then
$$\begin{aligned} &f_n(P \parallel \text{new } x \ Q) \ (P \parallel \text{new } x \ Q = \text{new } x \ (Q \parallel P)) : \\ &f_n(P \parallel \text{new } x \ Q) = f_n(P) \cup (f_n(Q) \setminus \{x\}) \stackrel{(*)}{=} \\ &(f_n(P) \cup f_n(Q)) \setminus \{x\} = f_n(\text{new } x \ (P \parallel Q)) \quad \checkmark \end{aligned}$$
8. $f_n(!P) = f_n(P) \cup f_n(P) = f_n(P \parallel !P) \quad \checkmark$