

Concurrency Theory
Series 6 - Solution

Exercise 1 Structural Congruence
Induction Base:

$$\begin{array}{c}
 (\tau) \frac{}{\underbrace{\tau.R + S}_P \rightarrow \underbrace{R}_Q} \\
 \\
 (\text{React}) \frac{}{\underbrace{(x(y).R + S) \parallel (\bar{x}\langle z \rangle.T + U)}_P \rightarrow \underbrace{R[z/y] \parallel T}_Q}
 \end{array}$$

Inductive Hypothesis (I.H): The assumption $(P \rightarrow Q)$ implies that there exists a derivation of this reduction in which the (Struct) rule is applied, if at all, only as the last rule.) holds for all subtrees.

Inductive Step:

$$(\text{Struct}) \frac{\frac{}{P \rightarrow Q}}{P' \rightarrow Q'}$$

By I.H. there exists a derivation tree

$$\begin{array}{c}
 (\text{Struct}) \frac{\frac{}{P'' \rightarrow Q''}}{P \rightarrow Q} \\
 (\text{Struct}) \frac{P \rightarrow Q}{P' \rightarrow Q'}
 \end{array}$$

$$P'' \equiv P, P' \equiv P, Q'' \equiv Q, Q' \equiv Q.$$

$$(\text{Struct}) \frac{\frac{}{P'' \rightarrow Q''}}{P' \rightarrow Q'}$$

... is a derivation tree satisfying the assumption.

$$(\text{Par}) \frac{\frac{}{P \rightarrow Q}}{P \parallel R \rightarrow Q \parallel R}$$

By I.H. there exists a derivation tree

$$\begin{array}{c}
 (\text{Struct}) \frac{\frac{}{P' \rightarrow Q'}}{P \rightarrow Q} \\
 (\text{Par}) \frac{P \rightarrow Q}{P \parallel R \rightarrow Q \parallel R}
 \end{array}$$

$$P' \equiv P, Q' \equiv Q \Rightarrow P' \parallel R \equiv P \parallel R, Q' \parallel R \equiv Q \parallel R.$$

$$\begin{array}{c}
 (\text{Par}) \frac{\frac{}{P' \rightarrow Q'}}{P' \parallel R \rightarrow Q' \parallel R} \\
 (\text{Struct}) \frac{P' \parallel R \rightarrow Q' \parallel R}{P \parallel R \rightarrow Q \parallel R}
 \end{array}$$

... is a derivation tree satisfying the assumption.

$$(\text{Res}) \frac{\frac{}{P \rightarrow Q}}{\text{new } x \ P \rightarrow \text{new } x \ Q}$$

By I.H. there exists a derivation tree

$$\begin{array}{c}
 (\text{Struct}) \frac{\frac{}{P' \rightarrow Q'}}{P \rightarrow Q} \\
 (\text{Res}) \frac{P \rightarrow Q}{\text{new } x \ P \rightarrow \text{new } x \ Q}
 \end{array}$$

$$P' \equiv P, Q' \equiv Q \Rightarrow \text{new } x \ P' \equiv \text{new } x \ P, \text{new } x \ Q' \equiv \text{new } x \ Q.$$

$$\begin{array}{c}
 (\text{Res}) \frac{\frac{}{P' \rightarrow Q'}}{\text{new } x \ P' \rightarrow \text{new } x \ Q'} \\
 (\text{Struct}) \frac{\text{new } x \ P' \rightarrow \text{new } x \ Q'}{\text{new } x \ P \rightarrow \text{new } x \ Q}
 \end{array}$$

... is a derivation tree satisfying the assumption.

Exercise 2 Reaction Relation

$$\begin{array}{l}
 S \equiv \text{new } x (\\
 \quad (x(u).u(y).u(z).\bar{y}\langle z \rangle.\text{nil}) \\
 \quad \parallel x(t).t(w).t(v).\bar{v}\langle w \rangle.\text{nil}) \\
 \quad \parallel \underbrace{\text{!new } s(\bar{x}\langle s \rangle.\bar{s}\langle a \rangle.\bar{s}\langle b \rangle.\text{nil})}_P \\
)
 \end{array}$$

$$\begin{aligned}
& \stackrel{*}{=} \text{new } x, s(\\
& \quad (x(u).u(y).u(z).\bar{y}\langle z \rangle.\text{nil} \\
& \quad || x(t).t(w).t(v).\bar{v}\langle w \rangle.\text{nil} \\
& \quad || \bar{x}\langle s \rangle.\bar{s}\langle a \rangle.\bar{s}\langle b \rangle.\text{nil} \\
& \quad || P \\
&)
\end{aligned}$$

$$\begin{aligned}
& \rightarrow^5 \text{new } x(\\
& \quad (\bar{a}\langle b \rangle.\text{nil} \\
& \quad || \bar{b}\langle a \rangle.\text{nil}) \\
& \quad || P \\
&) \\
& \equiv (\bar{a}\langle b \rangle.\text{nil} \\
& \quad || \bar{b}\langle a \rangle.\text{nil}) \\
& \quad || \text{new } x(!\text{new } s(\bar{x}\langle s \rangle.\bar{s}\langle a \rangle.\bar{s}\langle b \rangle.\text{nil}))
\end{aligned}$$

* 1.unfolding, 2.scope extension

$$\begin{aligned}
& \rightarrow \text{new } x, s(\\
& \quad (s(y).s(z).\bar{y}\langle z \rangle.\text{nil} \\
& \quad || x(t).t(w).t(v).\bar{v}\langle w \rangle.\text{nil} \\
& \quad || \bar{s}\langle a \rangle.\bar{s}\langle b \rangle.\text{nil} \\
& \quad || P \\
&) \\
& \rightarrow \text{new } x, s(\\
& \quad (s(z).\bar{a}\langle z \rangle.\text{nil} \\
& \quad || x(t).t(w).t(v).\bar{v}\langle w \rangle.\text{nil}) \\
& \quad || \bar{s}\langle b \rangle.\text{nil} \\
& \quad || P \\
&) \\
& \rightarrow \text{new } x, s(\\
& \quad (\bar{a}\langle b \rangle.\text{nil} \\
& \quad || x(t).t(w).t(v).\bar{v}\langle w \rangle.\text{nil}) \\
& \quad || \text{nil} \\
& \quad || P \\
&)
\end{aligned}$$

$$\begin{aligned}
& \stackrel{*}{=} \text{new } x, s(\\
& \quad (\bar{a}\langle b \rangle.\text{nil} \\
& \quad || x(t).t(w).t(v).\bar{v}\langle w \rangle.\text{nil}) \\
& \quad || \bar{x}\langle s \rangle.\bar{s}\langle a \rangle.\bar{s}\langle b \rangle.\text{nil} \\
& \quad || P \\
&)
\end{aligned}$$

* 1.discard nil (Abelian monoid laws for ||), 2.unfolding, 3. scope extension

Exercise 3 Polyadic π -Calculus

a)

$$\begin{aligned}
& x(y_1, y_2).P \parallel \bar{x}\langle z_1, z_2 \rangle.Q \parallel \bar{x}\langle z'_1, z'_2 \rangle.Q' \\
& \quad \downarrow \text{encoding} \\
& \overbrace{x(w).w(y_1).w(y_2).P}^{P_1} \parallel \overbrace{\text{new } w'(\bar{x}\langle w' \rangle.\bar{w}'\langle z_1 \rangle.\bar{w}'\langle z_2 \rangle.Q)}^{Q_1} \\
& \quad \parallel \underbrace{\text{new } w''(\bar{x}\langle w'' \rangle.\bar{w}''\langle z'_1 \rangle.\bar{w}''\langle z'_2 \rangle.Q')}_{Q_2} \\
& P_1 \parallel Q_1 \parallel Q_2 \\
& \equiv \text{new } w'(\\
& \quad x(w).w(y_1).w(y_2).P \\
& \quad || \bar{x}\langle w' \rangle.\bar{w}'\langle z_1 \rangle.\bar{w}'\langle z_2 \rangle.Q \\
&) \\
& || Q_2 \\
& \rightarrow \text{new } w'(\\
& \quad w'(y_1).w'(y_2).P \\
& \quad || \bar{w}'\langle z_1 \rangle.\bar{w}'\langle z_2 \rangle.Q \\
&) \\
& || Q_2 \\
& \rightarrow \text{new } w'(\\
& \quad w'(y_2).P[z_1/y_1] \\
& \quad || \bar{w}'\langle z_2 \rangle.Q \\
&) \\
& || Q_2 \\
& \rightarrow \text{new } w'(P[z_1/y_1, z_2/y_2] \parallel Q) \\
& || Q_2 \\
& \equiv P[z_1/y_1, z_2/y_2] \parallel Q \parallel Q_2 \\
& \rightarrow^* P[z'_1/y_1, z'_2/y_2] \parallel Q' \parallel Q_1
\end{aligned}$$