

Concurrency Theory
Series 7 - Solution

Exercise 1 Strong Bisimulation

1. $\mathcal{R} = \emptyset$
2. $\mathcal{R} = \{(P, P'), (Q, Q'), (R, R'), (R, R'')\}$
3. No, because $\mathcal{R}$ is not reflexive.

Exercise 2 Trace Equivalence and Strong Bisimulation

1. $\text{Tr}(P) = \epsilon + (ab^*c)^* + (ab^*c)ab^* = \text{Tr}(P')$
- 2.

$$\sim^0 = \text{Prc} \times \text{Prc}$$

$$\sim^1 = \text{id} \cup \{(Q, R), (R, Q), (Q, R'), (R', Q), (R, R'), (R', R), (P, P'), (P', P)\}$$

$$\sim^2 = \text{id} \cup \{(Q, R), (R, Q), (Q, R'), (R', Q), (R, R'), (R', R)\}$$

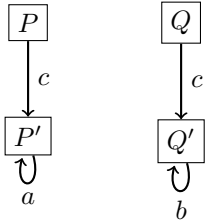
$$\sim^3 = \text{id} \cup \{(Q, R), (R, Q)\}$$

$$\sim^4 = \sim^3 \Rightarrow P \not\sim P'$$

Exercise 3 CCS Congruence

1. $P = a.P, \quad Q = a.Q + a.\text{nil}$
 $\text{TR}(P) = a^* = \text{TR}(Q)$
 $\text{CT}(P) = \emptyset \neq \{a\} = \text{CT}(Q)$

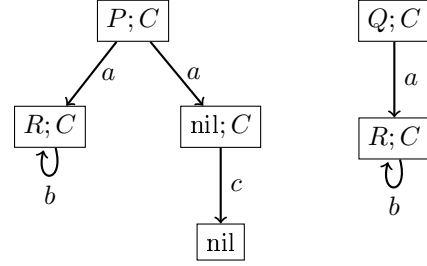
2. LTS for P, Q :



$$P \equiv_{\text{CT}} Q.$$

However, $\text{CT}(P \setminus \{a\}) = \{c\} \neq \emptyset = \text{CT}(Q \setminus \{a\})$
 $\Rightarrow$ CT-equivalence is not a congruence.

3. $C = c.\text{nil}$:



$$\text{TR}(P;C) = \epsilon + ab^* + ac, \quad \text{TR}(Q;C) = \epsilon + ab^* \\ \Rightarrow P;C \equiv_{\text{TR}} Q;C$$

Exercise 4 Coinduction

Let $F_A^\omega \times A^\omega \rightarrow A^\omega \times A^\omega$ be given by:
 $\forall \omega, v \in A^\omega$:

$$(\omega, v) \in F(\mathcal{R}) \Leftrightarrow \omega[0] = v[0] \text{ and } (\omega', v') \in \mathcal{R}$$

Then $\sim = \text{gfp}(F)$ (greatest fix-point).

Let $\underline{\mathcal{R}} := \{(\omega, \omega) \mid \omega \in A^\omega\}$. To show: $\underline{\mathcal{R}} = \sim$

1. " $\subseteq$ "

$$\begin{aligned} (\omega, v) \in F(\underline{\mathcal{R}}) &\Leftrightarrow \omega[0] = v[0] \text{ and } (\omega', v') \in \underline{\mathcal{R}} \\ &\Leftrightarrow \omega[0] = v[0] \text{ and } \omega' = v' \\ &\Leftrightarrow \omega = v \\ &\Leftrightarrow (\omega, v) \in \underline{\mathcal{R}} \end{aligned}$$

$$\Rightarrow F(\underline{\mathcal{R}}) = \underline{\mathcal{R}} \Rightarrow \underline{\mathcal{R}} \subseteq \sim = \text{gfp}(F)$$

2. " $\supseteq$ "

Define: $\omega \neq_k v \Leftrightarrow \omega = \omega_1 x \omega_2, v = v_1 y v_2, \omega_1 = v_1, |\omega_1| = |v_1| = k$ and $x, y \in A, x \neq y$. ($\omega \neq_k v = \omega \neq v$ an der Stelle k .)

Then $\omega \neq v \Leftrightarrow \exists k \in \mathbb{N} : \omega \neq_k v$.

Show by induction on k : $\forall k \in \mathbb{N} : \omega \neq_k v \Rightarrow \omega \not\sim v$

I.B.: $\omega \neq_0 v \Rightarrow \omega[0] \neq v[0] \Rightarrow \omega \not\sim v$.

I.H.: For a fixed $k \in \mathbb{N} : \omega \neq_k v \Rightarrow \omega \not\sim v \forall \omega, v \in A^\omega$.

I.S.: $\omega \neq_{k+1} v \Leftrightarrow \omega[0] = v[0] \text{ and } \omega' \neq_k v' \text{ } (\omega', v' = \text{words without first letter.}) \xrightarrow{I.H.} \omega' \not\sim v' \xRightarrow{\sim = \text{gfp}(F)} \omega \not\sim v$.

Hence, $\omega = v \Leftrightarrow (\omega, v) \in \mathcal{R} = \sim$.