

Concurrency Theory
Series 8 - Solution

Exercise 1 Game Characterization of Bisimulation

1. " $\Rightarrow$ ": $s \sim t$. Construct a universal winning strategy for the defender.
 - (a) Attacker chooses s and makes move $s \xrightarrow{\alpha} s'$. By $s \sim t$ there exists a t' so that $t \xrightarrow{\alpha} t'$ and $s' \sim t'$. Defender has a universal winning strategy from (s', t') .
 - (b) Attacker chooses t and makes move $t \xrightarrow{\alpha} t'$. By $s \sim t$ there exists a s' so that $s \xrightarrow{\alpha} s'$ and $s' \sim t'$. Defender has a universal winning strategy from (s', t') .
- " $\Leftarrow$ ": Defender has a universal winning strategy. Show that $s \sim t$ holds.
 - (a) If $s \xrightarrow{\alpha} s'$, then the defender can choose t' , so that $t \xrightarrow{\alpha} t'$ and the defender has a universal winning strategy from (s', t') .
 - (b) If $t \xrightarrow{\alpha} t'$, then the defender can choose s' , so that $s \xrightarrow{\alpha} s'$ and the defender has a universal winning strategy from (s', t') .
2. " $\Leftarrow$ ": Attacker has a universal winning strategy. Assume $s \sim t$ holds. $\Rightarrow$ Defender has a universal winning strategy. Both have a universal winning strategy from (s, t) . $\not\Rightarrow s \not\sim t$.

" $\Leftarrow$ ": $s \not\sim t$. There exists $s \xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_k} s'$. But t cannot mimick this. Choose that as winning strategy for the attacker.

Exercise 2 Monotonicity of $\mathcal{F}$

Let $\mathcal{R}, \mathcal{S} \in \mathcal{P}(\text{Prc} \times \text{Prc})$, $\mathcal{R} \supseteq \mathcal{S}$.

$$\mathcal{F}(\mathcal{R}) = \{(P, Q) \mid [P \xrightarrow{\alpha} P' \Rightarrow (\exists Q' : Q \xrightarrow{\alpha} Q' \wedge (P', Q') \in \mathcal{R})] \wedge [Q \xrightarrow{\alpha} Q' \Rightarrow (\exists P' : P \xrightarrow{\alpha} P' \wedge (P', Q') \in \mathcal{R})]\}$$

$$\mathcal{R} \supseteq \mathcal{S}$$

$$\begin{aligned} &\supseteq \{(P, Q) \mid [P \xrightarrow{\alpha} P' \Rightarrow (\exists Q' : Q \xrightarrow{\alpha} Q' \wedge (P', Q') \in \mathcal{S})] \\ &\quad \wedge [Q \xrightarrow{\alpha} Q' \Rightarrow (\exists P' : P \xrightarrow{\alpha} P' \wedge (P', Q') \in \mathcal{S})]\} \\ &= \mathcal{F}(\mathcal{S}) \end{aligned}$$

Exercise 3 Greatest Fixed Point Characterization of $\sim$

By Prop.: $\sim = \bigcup \{\mathcal{R} \in \mathcal{P}(\text{Prc} \times \text{Prc}) \mid \mathcal{R} \subseteq \mathcal{F}(\mathcal{R})\}$

$\mathcal{F}$ monotone on $(\mathcal{P}(\text{Prc} \times \text{Prc}), \supseteq) \Rightarrow \mathcal{F}$ monotone on $(\mathcal{P}(\text{Prc} \times \text{Prc}), \subseteq)$.

$$\begin{aligned} \text{gfp}(\mathcal{F}) &= \text{lub}\{\mathcal{R} \in \mathcal{P}(\text{Prc} \times \text{Prc}) \mid \mathcal{R} \subseteq \mathcal{F}(\mathcal{R})\} \\ &\quad (\text{Theorem 5.12, Tarski}) \\ &= \bigcup \{\mathcal{R} \in \mathcal{P}(\text{Prc} \times \text{Prc}) \mid \mathcal{R} \subseteq \mathcal{F}(\mathcal{R})\} \\ &= \sim \end{aligned}$$

Exercise 4 Strong Bisimilarity as a Game

$s \not\sim t$. Universal winning strategy for the attacker:

Attacker	Defender
$(s, t) \xrightarrow{a} (s, t_1)$	$(s, t_1) \xrightarrow{a} (s_1, t_1)$
$(s_1, t_1) \xrightarrow{b} (s_1, t_3)$	$(s_1, t_3) \xrightarrow{b} (s_2, t_3)$
$(s_2, t_3) \xrightarrow{b} (s_2, t_3)$	Defender is stuck.

Exercise 5 Ready Simulation and Bisimulation

$$P \equiv_{\text{rs}} Q, Q \equiv_{\text{rs}} P \Rightarrow P \sim Q.$$

Construct a universal winning strategy for the defender:

- If attacker plays $P \xrightarrow{\alpha} P'$, choose $Q \xrightarrow{\alpha} Q'$ from $P \equiv_{\text{rs}} Q$.
- If attacker plays $Q \xrightarrow{\alpha} Q'$, choose $P \xrightarrow{\alpha} P'$ from $Q \equiv_{\text{rs}} P$.

Always end up in a position where the attacker can α -move iff the defender can α -move.