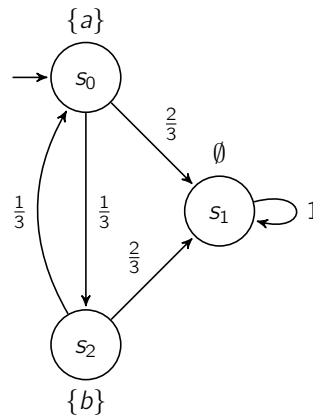


Exercise 1 (ω -regular Properties):

(4 points)

Non-deterministic Büchi automata are strictly more expressive than deterministic ones. Recall the “powerset construction”, which is used to compute a deterministic finite automaton from a non-deterministic one.

- Construct a non-deterministic Büchi automaton for the language $(a+b)^* a^\omega$, apply the powerset construction to determinize this automaton, and compare the languages of the resulting automaton and the original one by either showing their equivalence or giving a counterexample separating the languages.
- Consider the following DTMC D :



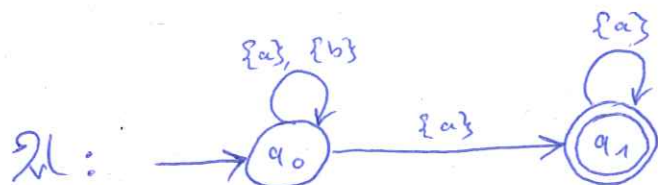
Give a formal definition for the cross-product between a *non-deterministic* Büchi-Automaton and a DTMC. Apply this definition to the DTMC D and the NBA from a). What problems arise?

Solution: _____

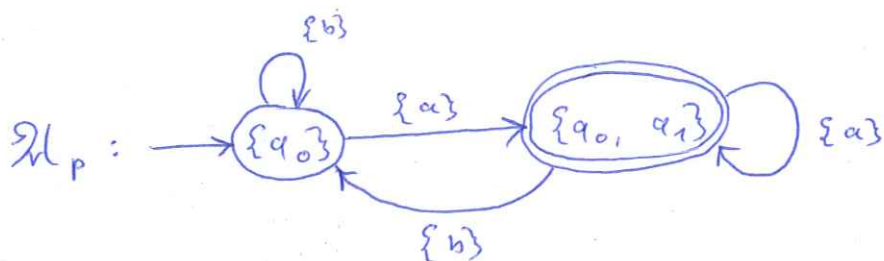
Series 3 - Solution

Exercise 1

a) NFA for $(a+b)^* a^\omega$:



Power set construction of $\mathcal{N}$:



Problem:

$(ab)^\omega \notin \mathcal{L}(\mathcal{N})$, but

$(ab)^\omega \in \mathcal{L}(\mathcal{N}_p)$ ⚡

b) Let $\mathcal{D} = (S, P, c_{init}, AP, L)$ be a DTMC and let $\mathcal{A} = (Q, \mathcal{P}(AP), \Delta, q_0, F)$ be an NBA.

Then $\mathcal{D} \boxtimes \mathcal{A} := (S \times Q, P', c'_{init}, \{accept\}, L')$ where:

$$P'((s, q), (s', q')) = \begin{cases} P(s, s'), & \text{if } (q, L(s')q') \in \Delta \\ 0, & \text{else} \end{cases}$$

$$c'_{init}((s, q)) = \begin{cases} c_{init}(s), & \text{if } (q_0, L(s), q) \in \Delta \\ 0, & \text{else} \end{cases}$$

$$L'((s, q)) = \begin{cases} \{accept\}, & \text{if } q \in F \\ \emptyset, & \text{else} \end{cases}$$

□

Now consider $\mathcal{D} \boxtimes \mathcal{A}$:

Both $c'_{init}((s_0, q_0)) = 1$ and $c'_{init}((s_0, q_1)) = 1$ $\Downarrow$

Exercise 2 (Probabilities vs. Qualitative Properties):

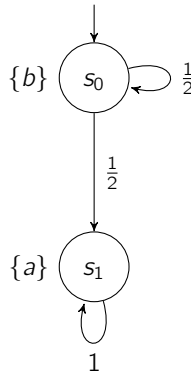
(3 points)

Let $D = (S, \mathbf{P}, s_{init}, AP, L)$ be a finite DTMC, $s \in S$, $a, b \in AP$. For each of the following statements, explain informally whether it is correct or not. If it is not correct, give a counterexample and indicate which of the implications (if any) hold.

- a) $Pr(s \models \Box a) = 1$ if and only if $s \models \forall \Box a$
- b) $Pr(s \models \Diamond a) < 1$ if and only if $s \not\models \forall \Diamond a$
- c) $Pr(s \models \Box a) > 0$ if and only if $s \models \exists \Box a$

Solution: _____

- a) The statement is true. The implication from the right-hand side to the left hand side is obvious. In order to show the other direction, we show $s \not\models \forall \Box a$ implies $Pr(s \models \Box a) < 1$. Since the first part means that there exists a finite path $\hat{\pi}$ that eventually reaches a state $s' \not\models a$, we have that $Pr(s \models \Box a) \leq 1 - Pr(\hat{\pi}) < 1$.
- b) This statement is not true. Consider the following DTMC.



Clearly, $s_0 \not\models \forall \Diamond a$ since there the path s_0^ω never reaches a state labeled with a . However, $Pr(s_0 \models \Diamond a) = 1$. The implication from left to right does indeed hold, because $s \models \forall \Diamond a$ implies that $Pr(s_0 \models \Diamond a) = 1$.

- c) This statement is not correct. Consider the DTMC from task b). Then $s_0^\omega \models \Box b$, but $Pr(s_0 \models \Box b) = 0$. Similarly to task b), the implication from left to right holds.

Exercise 3 (Algorithms for Qualitative Model Checking):

(3 points)

Let $D = (S, \mathbf{P}, s_{init}, AP, L)$ be a finite DTMC and $a, b \in AP$. Give algorithms (in pseudocode) to compute the following sets and briefly explain the complexity of your algorithm.

- a) $S_{=0} = \{s \in S \mid Pr(s \models aUb) = 0\}$
- b) $S_{=1} = \{s \in S \mid Pr(s \models aUb) = 1\}$

Solution: _____

- a)
- ```
 $R \leftarrow \emptyset$
 $R' \leftarrow \text{Sat}(b)$
while ($R \neq R'$) {
 $R \leftarrow R'$
 $R' \leftarrow R \cup \{s \in \text{Sat}(a) \mid \exists s' \in R. P(s, s') > 0\}$
}
return $S \setminus R$;
```
- b)
- ```
 $R \leftarrow \emptyset$   
 $R' \leftarrow S_{=0}$   
while ( $R \neq R'$ ) {  
     $R \leftarrow R'$   
     $R' \leftarrow R \cup \{s \in (\text{Sat}(a) \setminus \text{Sat}(b)) \mid \exists s' \in R. P(s, s') > 0\}$   
}  
return  $S \setminus R$ ;
```