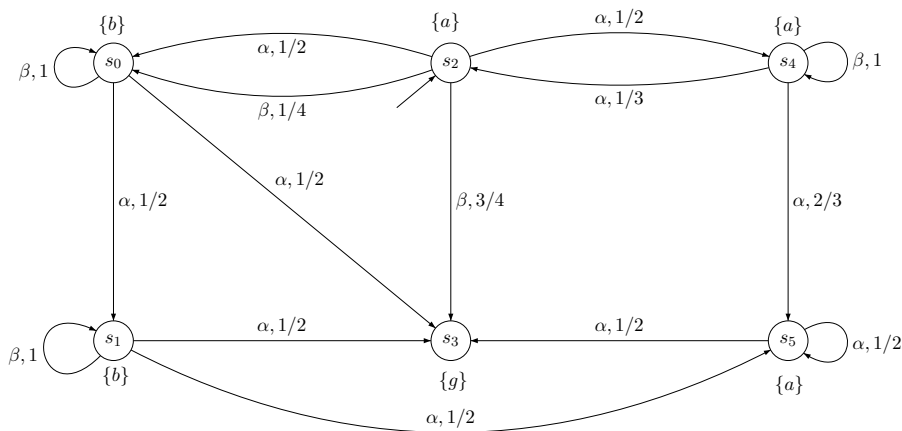


Exercise 1 (PCTL Model Checking):

(5 points)



Consider the MDP M as shown above. Find out the satisfaction sets of the following properties or list the probabilities corresponding to a maximizing scheduler for all states, give the used policies, and describe their properties:

1. $\mathbb{P}_{\geq 0.5}(\bigcirc a)$
2. $\mathbb{P}_{\geq .6}(\bigcirc b)$
3. $\mathbb{P}_{\leq .3}(\Diamond g)$
4. $\Pr^M(s \models (\Box \Diamond a))$
5. $\Pr^M(s \models (\Box \Diamond a \wedge \neg \Box \Diamond b))$

Solution: _____

1. Sat:

s_1, s_2, s_4, s_5

Pos. Policy:

$-, \alpha, \alpha, -, \beta, \alpha$

2. Sat:

s_0, s_1

Pos. Policy:

$\beta, \beta, -, -, -, -$

3. Sat:

s_0, s_1, s_2, s_4

Pos. Policy:

$\beta, \beta, \alpha, -, \beta, \alpha$

4.

$0, 0, \frac{1}{2}, 0, 1, 0$

Pos. Policy:

$\alpha, \alpha, \alpha, -, \beta, \alpha$

5.

$$0, 0, \frac{1}{2}, 0, 1, 0$$

Exercise 2 (Exponential Distribution):

(3 points)

Show that the maximum of two exponential distributions is not an exponential distribution.

Solution: _____

$$\begin{aligned}
 \Pr(\max(X, Y) > t) &= \Pr(X > t \vee Y > t) \\
 &= 1 - \Pr(X \leq t \wedge Y \leq t) \\
 &= 1 - \Pr(X \leq t) \cdot \Pr(Y \leq t) \\
 &= 1 - (1 - \Pr(X > t))(1 - \Pr(Y > t)) \\
 &= 1 - (1 - \Pr(X > t) - \Pr(Y > t) + \Pr(X > t) \Pr(Y > t)) \\
 &= \Pr(X > t) + \Pr(Y > t) - \Pr(X > t) \Pr(Y > t) \\
 &= e^{-\lambda t} + e^{-\mu t} - e^{-(\lambda+\mu)t}
 \end{aligned}$$

Exercise 3 (CTMC Modelling):

(2 points)

We consider a server system consisting of two servers and a queue of capacity one. Jobs arrive with a rate λ , and are scheduled to any available server. If no server is available, they're put in the queue. The servers handle the jobs with rate μ . If done, a job from the queue is taken, if there is one. Otherwise, the server is idle. Model the system as a CTMC.

Solution: _____

