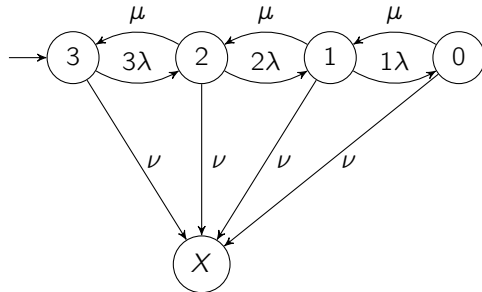


Exercise 1 (CTMC Model Checking):

(4 points)

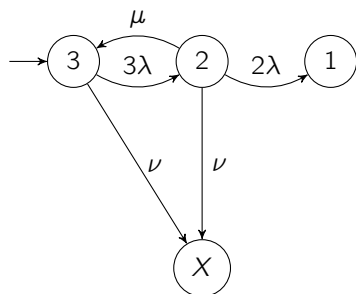
Consider the Example CTMC from the lecture (displayed below).



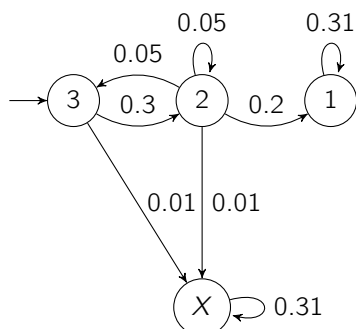
- Compute $Pr(\Diamond^{\leq 3}\{1\})$ using $\lambda = \frac{1}{10}$, $\mu = \frac{1}{20}$ and $\nu = \frac{1}{100}$. You may cut off an infinite sum after three terms.
- Give the closed form expression for $Pr(\Diamond^{\leq t}\{X\})$. *Hint: The closed form is not that large.*

Solution: _____

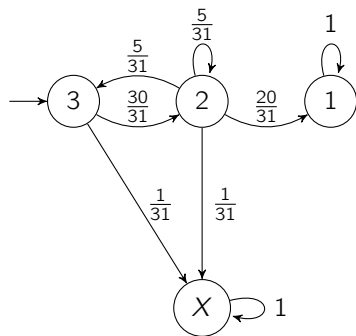
- First, we make the final states absorbing. This yields the following CTMC (only reachable states are relevant).



Next, we do the substitution of the variables and the uniformization with rate 31/100. This yields the following.



The embedded DTMC is given by:



Compute

$$p(0) \cdot e^{-3 \cdot 0.31} \cdot \sum_{i=0}^{\infty} \frac{(0.31 \cdot 3)^i}{i!} P^i$$

where P is the transition matrix of the embedded DTMC.

The final probability is given by the vector-entry corresponding to the state 2.

b) First, we apply weak-bisimulation. This yields a two state CTMC. We then obtain from the definition $e^{-\nu \cdot t}$.

Exercise 2 (The Erlang distribution):

(2 points)

The cumulative distribution function (CDF) of an *Erlang distribution* with *shape* k and *rate* λ can be written as

$$F(t; k, \lambda) = 1 - \sum_{n=0}^{k-1} \frac{1}{n!} e^{-\lambda \cdot t} (\lambda \cdot t)^n$$

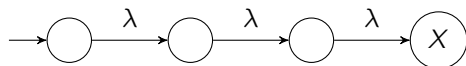
Construct a CTMC such that the transient probability for state x coincides with the CDF for the Erlang distribution with $k = 3$ and $\lambda = 4$. Briefly explain your construction!

Solution:

We observe that the formula resembles the solution for uniformized CTMCs in several ways. We look for a solution such that:

- The graph is already uniformized.
- There seems to be no branching, as all transition probabilities should be equal and thus 1.
- We are only interested in graphs where the longest (relevant) prefix has length $k - 1$.

Solution:



Exercise 3 (CTMC Model Checking for $I = (t_1, t_2)$):

(4 points)

- Show that for a CTMC with states S and $T \subset S$ and $0 < t_1 < t_2$ the following equation does **not** hold:
 $\Pr(\Diamond^{(t_1, t_2)} T) = \Pr(\Diamond^{\leq t_2} T) - \Pr(\Diamond^{\leq t_1} T)$.
- Sketch an algorithm to compute for a given CTMC with states S and $T \subset S$ and $0 < t_1 < t_2$ the following property: $\Pr(\Diamond^{(t_1, t_2)} T)$

Solution: _____

- a) As a counterexample, consider a DTMC with just one state. Any reachability yields one, however one minus one = zero.
- b) The algorithm works by a two step approach.
 - a) Compute the transient probabilities for t_1 .
 - b) Compute the reachability probabilities for $t_2 - t_1$, using the result of the first step as initial distribution.