

Exersice Sheet 3

———— Sample Solution ————

Task 1: Operational Equivalence

Prove or disprove:

repeat c **until** $b \sim c$; **while** b **do** c **end**

The claim will be disproved using a counter example.

Lets assume $b := \mathbf{true}$ and $c := \mathbf{skip}$.

The **repeat** statement will terminate after the first iteration with $\langle \mathbf{repeat} \ c \ \mathbf{until} \ b, \sigma \rangle \rightarrow \sigma$ while the **while** statement will never terminate as its condition is always satisfied.

Prove or disprove:

$\underbrace{\mathbf{repeat} \ c \ \mathbf{until} \ b \sim c}_{=r}; \underbrace{\mathbf{while} \ \neg b \ \mathbf{do} \ c \ \mathbf{end}}_{=w}$

To prove this statement we claim that

$\langle r, \sigma \rangle \rightarrow \sigma' \Leftrightarrow \langle c; w, \sigma \rangle \rightarrow \sigma'$

” $\Rightarrow$ ”

Proof by induction on the structure of proof trees

$\langle \mathbf{repeat} \ c \ \mathbf{until} \ b, \sigma \rangle \rightarrow \sigma'$

Induction Base: ($b = \mathbf{true}$)

$$\frac{\frac{\dots}{\langle c, \sigma \rangle \rightarrow \sigma'} \quad \frac{\dots}{\langle b, \sigma 1' \rangle \rightarrow \mathbf{true}}}{\langle r, \sigma \rangle \rightarrow \sigma'} \text{ (repeat-true)}$$

Applying this assignment to the **while**-sequence we receive:

$$\begin{array}{c}
\frac{\langle b, \sigma' \rangle \rightarrow \mathbf{true}}{\langle \neg b, \sigma' \rangle \rightarrow \mathbf{false}} \\
\frac{\langle c, \sigma \rangle \rightarrow \sigma' \quad \frac{\langle \neg b, \sigma' \rangle \rightarrow \mathbf{false}}{\langle w, \sigma' \rangle \rightarrow \sigma'} \text{ (while-false)}}{\langle c; w, \sigma \rangle \rightarrow \sigma'} \text{ (seq)}
\end{array}$$

Thus the claim holds if $b = \mathbf{true}$.

Induction Hypothesis:

$\langle r, \sigma' \rangle \rightarrow \sigma'' \Rightarrow \langle c; w, \sigma' \rangle \rightarrow \sigma''$ holds.

Induction step: ($b = \mathbf{false}$)

$$\frac{\frac{\text{assumption}}{\langle c, \sigma \rangle \rightarrow \sigma'} \quad \langle b, \sigma' \rangle \rightarrow \mathbf{false} \quad \frac{\text{by assumption}}{\langle r, \sigma' \rangle \rightarrow \sigma''}}{\langle r, \sigma \rangle \rightarrow \sigma''} \text{ (repeat-false)}$$

Applying the same assignment to the **while**-sequence we receive:

$$\frac{\frac{\text{assumption}}{\langle c, \sigma \rangle \rightarrow \sigma'} \quad \frac{\frac{\langle b, \sigma' \rangle \rightarrow \mathbf{false}}{\langle \neg b, \sigma' \rangle \rightarrow \mathbf{true}} \quad \frac{\frac{\text{I.H.}}{\langle c; w, \sigma' \rangle \rightarrow \sigma''} \quad \langle c, \sigma' \rangle \rightarrow \sigma''' \quad \langle w, \sigma''' \rangle \rightarrow \sigma''}{\langle w, \sigma' \rangle \rightarrow \sigma''}}{\langle c; w, \sigma \rangle \rightarrow \sigma''} !$$

Thus the claim also holds for $b = \mathbf{false}$.

" $\Leftarrow$ " analogously.

Task 2: Translation of Statements

$$\mathfrak{T}_c[\text{repeat } c \text{ until } b] = \mathfrak{T}_c[c]; \mathfrak{T}_b[b]; \mathbf{JMPF}(-|\mathfrak{T}_c[c]| + |\mathfrak{T}_b[b]|)$$

Task 3: loop Loops

(a)

$$\frac{\frac{\langle x > 0, \sigma \rangle \rightarrow \mathbf{false}}{\langle \text{loop } x \text{ begin } c \text{ end}, \sigma \rangle \rightarrow \sigma} \quad \langle x > 0, \sigma \rangle \rightarrow \mathbf{true} \quad \langle c, \sigma \rangle \rightarrow \sigma' \quad \langle z := x - 1, \sigma' \rangle \rightarrow \sigma'' \quad \langle \text{loop } z \text{ begin } c \text{ end}, \sigma'' \rangle \rightarrow \sigma'''}{\langle \text{loop } x \text{ begin } c \text{ end}, \sigma \rangle \rightarrow \sigma'''}$$

(b)

$\mathfrak{T}_c[\text{loop } x \text{ begin } c \text{ end}] = \text{LOAD}(x); \text{STO}(\xi);$
 $\text{LOAD}(\xi); \text{PUSH}(0); \text{GT}; \text{JMPF}(|\mathfrak{T}_c[c]| + 6);$
 $\mathfrak{T}_c[c]; \text{LOAD}(\xi); \text{PUSH}(1); \text{SUB}; \text{STO}(\xi); \text{JMP}(-(|\mathfrak{T}_c[c]| + 8))$