

Exersice Sheet 7

Sample Solution

Task 1: Partial Correctness

(a)

```
p := 2;
q := n - p;
while ¬prime(p) ∨ ¬prime(q) do
    p := p + 1;
    q := n - p;
end
```

(b)

```
Let c = while ¬prime(p) ∨ ¬prime(q) do
    p := p + 1;
    q := n - p;
end
```

$$\text{(seq)} \frac{\text{(a)} \quad \text{(seq)} \frac{\text{(b)} \quad \text{(c)} \quad \overline{\{p = 2\} \quad q = n - p; \quad c \quad \{\mathbf{prime}(p) \wedge \mathbf{prime}(q) \wedge n = q + p\}}}{\{\mathbf{true}\} \quad p := 2; \quad q = n - p; \quad c \quad \{\mathbf{prime}(p) \wedge \mathbf{prime}(q) \wedge n = q + p\}}}{\{\mathbf{true}\} \quad p := 2; \quad q = n - p; \quad c \quad \{\mathbf{prime}(p) \wedge \mathbf{prime}(q) \wedge n = q + p\}}$$

(a)

$$\text{(cons)} \frac{\models (\mathbf{true} \Rightarrow 2 = 2) \quad \text{(asgn)} \quad \overline{\{2 = 2\} \quad p := 2; \quad \{p = 2\}} \quad \models (p = 2 \Rightarrow p = 2)}{\{\mathbf{true}\} \quad p := 2; \quad \{p = 2\}}$$

(b)

$$\text{(cons)} \frac{\models (p = 2 \Rightarrow n - p = n - p) \quad \text{(asgn)} \quad \overline{\{n - p = n - p\} \quad q := n - p; \quad \{q = n - p\}} \quad \models (q = n - p \Rightarrow q = n - p)}{\{p = 2\} \quad q = n - p; \quad \{q = n - p\}}$$

(c)

$$\text{(cons)} \frac{\models (q = n - p \Rightarrow n = q + p) \quad \text{(while)} \quad \frac{\text{(d)} \quad \overline{\{n = q + p\} \quad c \quad \{\mathbf{prime}(p) \wedge \mathbf{prime}(q) \wedge n = q + p\}} \quad \models ((\mathbf{prime}(p) \wedge \mathbf{prime}(q) \wedge n = q + p) \Rightarrow (\mathbf{prime}(p) \wedge \mathbf{prime}(q) \wedge n = q + p))}{\{q = n - p\} \quad c \quad \{\mathbf{prime}(p) \wedge \mathbf{prime}(q) \wedge n = q + p\}}}{\{q = n - p\} \quad c \quad \{\mathbf{prime}(p) \wedge \mathbf{prime}(q) \wedge n = q + p\}}$$

(d)

$$\text{(cons)} \frac{\models (n = q + p \wedge (\neg \mathbf{prime}(p) \vee \neg \mathbf{prime}(q)) \Rightarrow p + 1 = p + 1) \quad \text{(seq)} \quad \frac{\text{(asgn)} \quad \overline{\{p + 1 = p + 1\} \quad p := p + 1 \quad \{p = p + 1\}} \quad \text{(e)} \quad \overline{\{p + 1 = p + 1\} \quad p := p + 1; \quad q := n - p; \quad \{n = q + p\}} \quad \models (n = q + p \Rightarrow n = q + p)}{\{n = q + p \wedge (\neg \mathbf{prime}(p) \vee \neg \mathbf{prime}(q))\} \quad p := p + 1; \quad q := n - p; \quad \{n = q + p\}}$$

(e)

$$\text{(cons)} \frac{\models (p = p + 1 \Rightarrow n - p = n - p) \quad \text{(asgn)} \quad \overline{\{n - p = n - p\} \quad q := n - p \quad \{q = n - p\}} \quad \models (q = n - p \Rightarrow n = q + p)}{\{p = p + 1\} \quad q := n - p \quad \{n = q + p\}}$$

Task 2: Total Correctness

(a)

$$\text{(cons)} \frac{\models ((\mathbf{true} \wedge x \neq 0) \Rightarrow \mathbf{true}) \quad \overline{\{\mathbf{true}\} \quad x := x - 1 \quad \{\mathbf{true}\}} \quad \models (\mathbf{true} \Rightarrow \mathbf{true})}{\text{(cons)} \frac{\models (\mathbf{true} \Rightarrow \mathbf{true}) \quad \text{(while)} \quad \frac{\overline{\{\mathbf{true} \wedge x \neq 0\} \quad x := x - 1 \quad \{\mathbf{true}\}}}{\{\mathbf{true}\} \quad \mathbf{while} \quad x \neq 0 \quad \mathbf{do} \quad x := x - 1 \quad \mathbf{end} \quad \{\mathbf{true} \wedge x = 0\}} \quad \models ((x = 0 \wedge \mathbf{true}) \Rightarrow \mathbf{true})}{\{\mathbf{true}\} \quad \mathbf{while} \quad x \neq 0 \quad \mathbf{do} \quad x := x - 1 \quad \mathbf{end} \quad \{\mathbf{true}\}}}$$

(b)

$$\text{(cons)} \frac{\models (x > 0 \Rightarrow (\exists i \geq 0 \wedge x = i)) \quad \text{(while)} \quad \frac{\models ((i \geq 0 \wedge x = i + 1) \Rightarrow x \neq 0) \quad \text{(cons)} \quad \frac{\models ((i \geq 0 \wedge x = i + 1) \Rightarrow x = i + 1) \quad \overline{\{x = i + 1\} \quad x := x - 1 \quad \{\Downarrow x = i\}} \quad \models (x = i \Rightarrow x = i)}{\{i \geq 0 \wedge x = i + 1\} \quad x := x - 1 \quad \{\Downarrow x = i\}} \quad (x = 0 \Rightarrow x = 0)}{\{i \geq 0 \wedge x = i\} \quad \mathbf{while} \quad x \neq 0 \quad \mathbf{do} \quad x := x - 1 \quad \mathbf{end} \quad \{\Downarrow x = 0\}} \quad (x = 0 \Rightarrow \mathbf{true})}{\{x > 0\} \quad \mathbf{while} \quad x \neq 0 \quad \mathbf{do} \quad x := x - 1 \quad \mathbf{end} \quad \{\Downarrow \mathbf{true}\}}$$

(c)

This statement will be disproved by contradiction.

$$\begin{array}{c} \text{(cons)} \frac{\vdash ((x < 0 \wedge x \neq 0) \Rightarrow x < 0) \quad \text{(cons)} \frac{\frac{\vdash (x < 0 \Rightarrow x < 1) \quad \overline{\{x < 1\} \ x := x - 1 \ \{x < 0\}} \quad \vdash (x < 0 \Rightarrow x < 0)}{\{x < 0\} \ x := x - 1 \ \{x < 0\}} \quad \vdash (x < 0 \Rightarrow x < 0)}{\vdash ((x < 0 \wedge x \neq 0) \Rightarrow x < 0)} \\ \text{(while)} \frac{\{x < 0 \wedge x \neq 0\} \ x := x - 1 \ \{x < 0\}}{\{x < 0\} \ \mathbf{while} \ x \neq 0 \ \mathbf{do} \ x := x - 1 \ \mathbf{end} \ \{x < 0 \wedge x = 0\}} \\ \text{(cons)} \frac{\vdash (x < 0 \Rightarrow x < 0) \quad \text{(while)} \frac{\{x < 0 \wedge x \neq 0\} \ x := x - 1 \ \{x < 0\}}{\{x < 0\} \ \mathbf{while} \ x \neq 0 \ \mathbf{do} \ x := x - 1 \ \mathbf{end} \ \{x < 0 \wedge x = 0\}} \quad \vdash ((x = 0 \wedge x < 0) \Rightarrow \mathbf{false})}{\{x < 0\} \ \mathbf{while} \ x \neq 0 \ \mathbf{do} \ x := x - 1 \ \mathbf{end} \ \{\mathbf{false}\}} \end{array}$$

As this statement holds the total correctness of $\{\mathbf{true}\} \ \mathbf{while} \ x \neq 0 \ \mathbf{do} \ x := x - 1 \ \mathbf{end} \ \{\Downarrow \mathbf{true}\}$ is not satisfied.