

Exercise Sheet 6

———— Sample Solution ————

Task 1: Equivalence of Statements in ParWHILE

In ParWHILE it is not generally possible to replace program statements by equivalent ones without changing the programs behaviour.

Example:

$c_1 : \quad x := 1; x := x + 1$

$c_2 : \quad x := 2$

$c_3 : \quad x := 3$

Obviously, we have $c_1 \approx c_2$.

However, $c_1 \parallel c_3 \not\approx c_2 \parallel c_3$!

$\langle c_1 \parallel c_3, \sigma \rangle \rightarrow^* \sigma [x \mapsto k]$, where $k \in \{2, 3, 4\}$, but

$\langle c_2 \parallel c_3, \sigma \rangle \rightarrow^* \sigma [x \mapsto k]$, where $k \in \{2, 3\}$

Task 2: Fairness in CSP

Reminder:

An execution $k_0 \rightarrow k_1 \rightarrow \dots$, where $k_j = \langle c_1^{(j)} \parallel \dots \parallel c_n^{(j)}, \sigma \rangle$, is ...

(for some $1 \leq i \leq n$, $l_0 \in \mathbb{N}$, $c_i^{(l)} = c_i^{(l_0)} \forall l \geq l_0$)

- strongly unfair if $c_i^{(l)}$ is enabled in k_l for all $l \geq l_0$.
- weakly unfair if $c_i^{(l)}$ is enabled in k_l for infinitely many $l \geq l_0$.

We now prove that every strongly unfair execution is also weakly unfair:

Let $k = k_1 \rightarrow k_2 \dots$ be a strongly unfair execution with

$k_i = \langle c_1^{(i)} \parallel \dots \parallel c_n^{(i)}, \sigma_i \rangle$.

Then there exists $i \in [1, n]$ and $l_0 \in \mathbb{N}$ such that c_i^l is enabled for all $l \geq l_0$.

Since k is infinite this implies that c_i^l is enabled for infinitely many $l \geq l_0$.

Task 3: Axiomatic Semantics of Nondeterminism

(a)

$$\frac{\{P\} \ c_1 \ \{Q\} \quad \{P\} \ c_2 \ \{Q\}}{\{P\} \ c_1 \sqcup c_2 \ \{Q\}}$$

(b)

$$wp(c_1 \sqcup c_2, Q) = wp(c_1, Q) \wedge wp(c_2, Q)$$

(c)

Counterexample:

$$\begin{aligned} c : \quad & x := 0 \sqcup x := 1 \\ Q_1 : \quad & x = 0 \\ Q_2 : \quad & x = 1 \end{aligned}$$

We have

$$\begin{aligned} & wp(c, Q_1 \vee Q_2) \\ = & wp(x := 0, Q_1 \vee Q_2) \wedge wp(x := 1, Q_1 \vee Q_2) \\ = & (\mathbf{true} \vee \mathbf{false}) \wedge (\mathbf{false} \vee \mathbf{true}) \\ = & \mathbf{true} \wedge \mathbf{true} \\ = & \mathbf{true} \end{aligned}$$

However,

$$\begin{aligned} & wp(c, Q_1) \vee wp(c, Q_2) \\ = & (wp(x := 0, Q_1) \wedge wp(x := 1, Q_1)) \vee (wp(x := 0, Q_2) \wedge wp(x := 1, Q_2)) \\ = & (\mathbf{true} \wedge \mathbf{false}) \vee (\mathbf{false} \wedge \mathbf{true}) \\ = & \mathbf{false} \vee \mathbf{false} \\ = & \mathbf{false} \end{aligned}$$

(d)

$$\frac{\{P\} \ c_1 \ \{Q\}}{\{P\} \ c_1 \sqcup c_2 \ \{Q\}} \quad \frac{\{P\} \ c_2 \ \{Q\}}{\{P\} \ c_1 \sqcup c_2 \ \{Q\}}$$

(e)

$$wp(c_1 \sqcup c_2, Q) = wp(c_1, Q) \vee wp(c_2, Q)$$

(f)

Counterexample:

$$c : \quad x := 0 \Box x := 1$$

$$Q_1 : \quad x = 0$$

$$Q_2 : \quad x = 1$$

We have

$$\begin{aligned} & wp(c, Q_1 \wedge Q_2) \\ = & wp(x := 0, Q_1 \wedge Q_2) \vee wp(x := 1, Q_1 \wedge Q_2) \\ = & (\mathbf{true} \wedge \mathbf{false}) \vee (\mathbf{false} \wedge \mathbf{true}) \\ = & \mathbf{false} \vee \mathbf{false} \\ = & \mathbf{false} \end{aligned}$$

However,

$$\begin{aligned} & wp(c, Q_1) \wedge wp(c, Q_2) \\ = & (wp(x := 0, Q_1) \vee wp(x := 1, Q_1)) \wedge (wp(x := 0, Q_2) \vee wp(x := 1, Q_2)) \\ = & (\mathbf{true} \vee \mathbf{false}) \wedge (\mathbf{false} \vee \mathbf{true}) \\ = & \mathbf{true} \wedge \mathbf{true} \\ = & \mathbf{true} \end{aligned}$$