

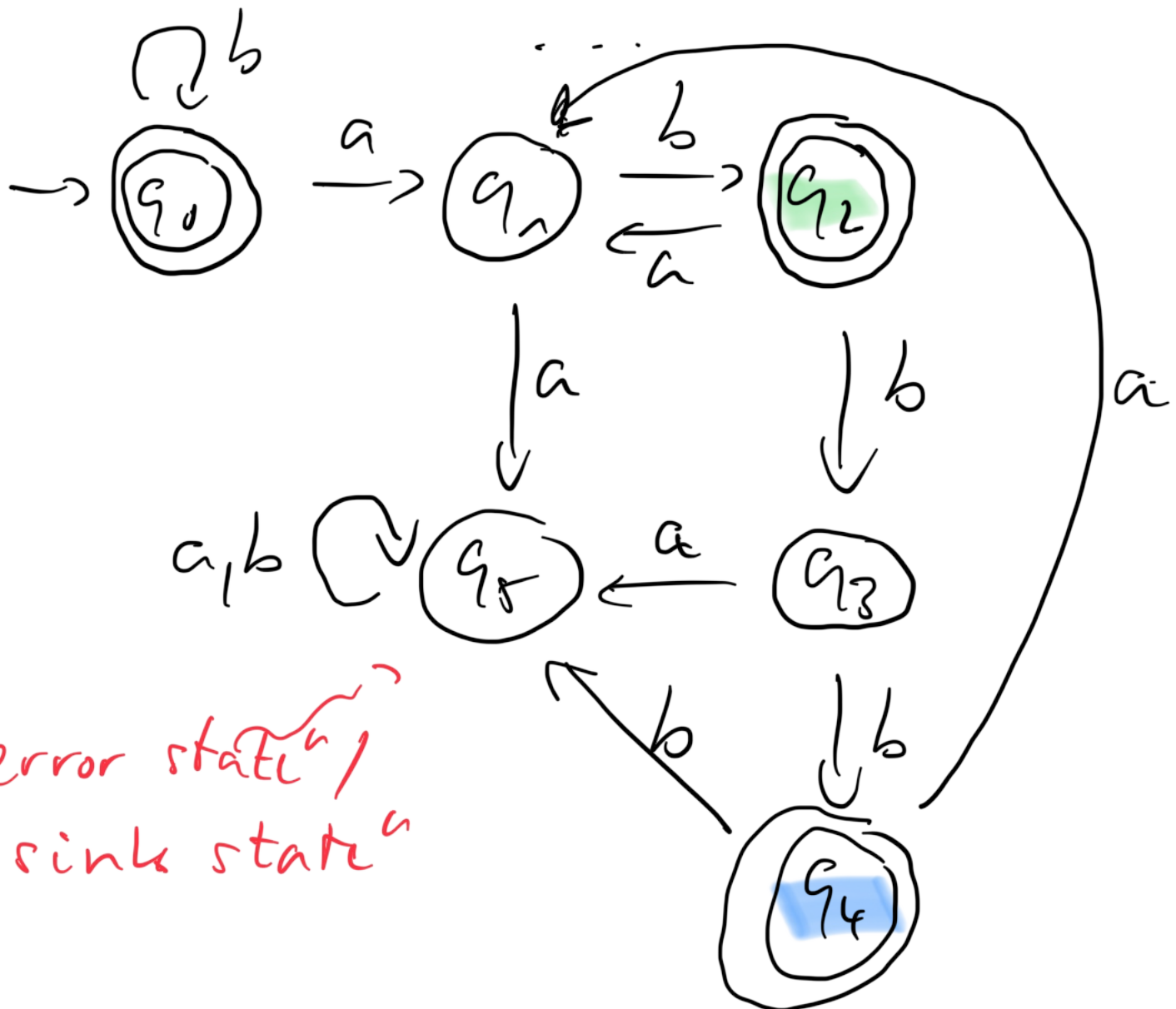
Exercises (Regular Languages)

A1: Construction of Deterministic Finite Automata

Task: Construct a DFA over $\Sigma := \{a, b\}$ that accepts the following language:

$$\{w \in \Sigma^* \mid \text{each } a \text{ followed by exactly 1 or 3 } b\text{'s}\}$$

Inputs: $\underline{\Sigma}$, $\underline{a}$, $\underline{b}$, $\underline{aa}$, $\underline{ab}$, $\underline{ba}$, $\underline{bb}$



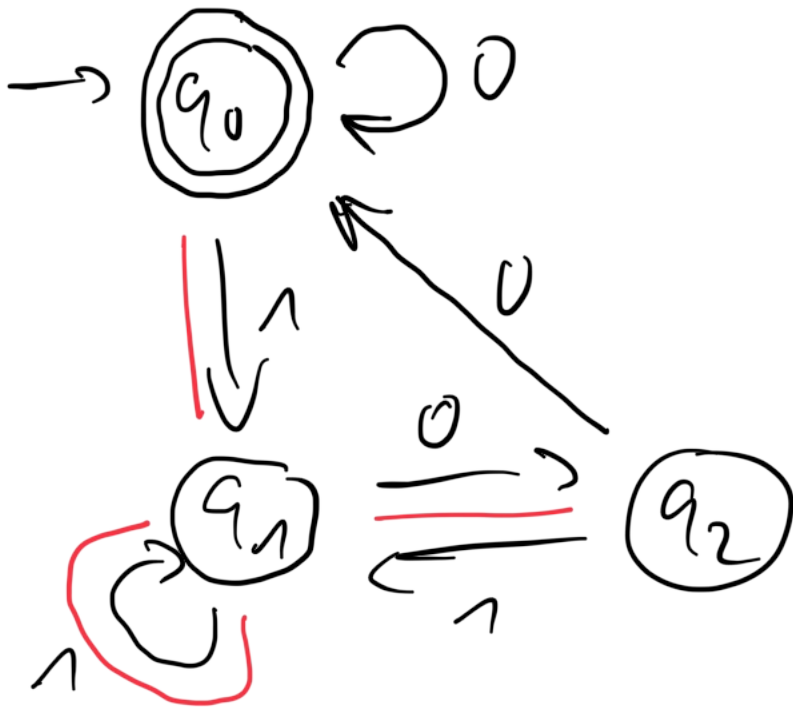
A2: Construction of Deterministic Finite Automata

Task: Construct a DFA over $\Sigma := \{0, 1\}$ that accepts the following language:

$$\{w \in \Sigma^* \mid \text{decimal value of } w \text{ divisible by } 4\}$$

Accepted: $\Sigma, 0, 1, 00, 01, 10, 11, 000, 001, \dots$

$\Rightarrow$ equivalent: w is of the form $\Sigma, 0, 00$, or $v00$ ($v \in \Sigma^*$).

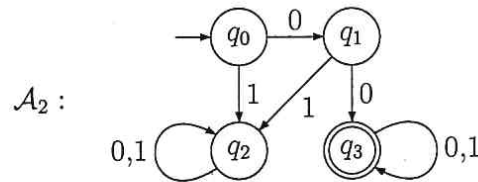
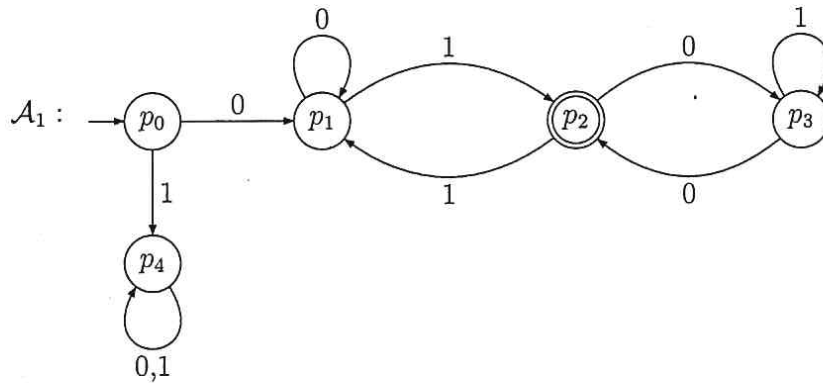


$$110 \hat{=} 6 \\ \Rightarrow q_2 \notin K$$

$$011 \hat{=} 3$$

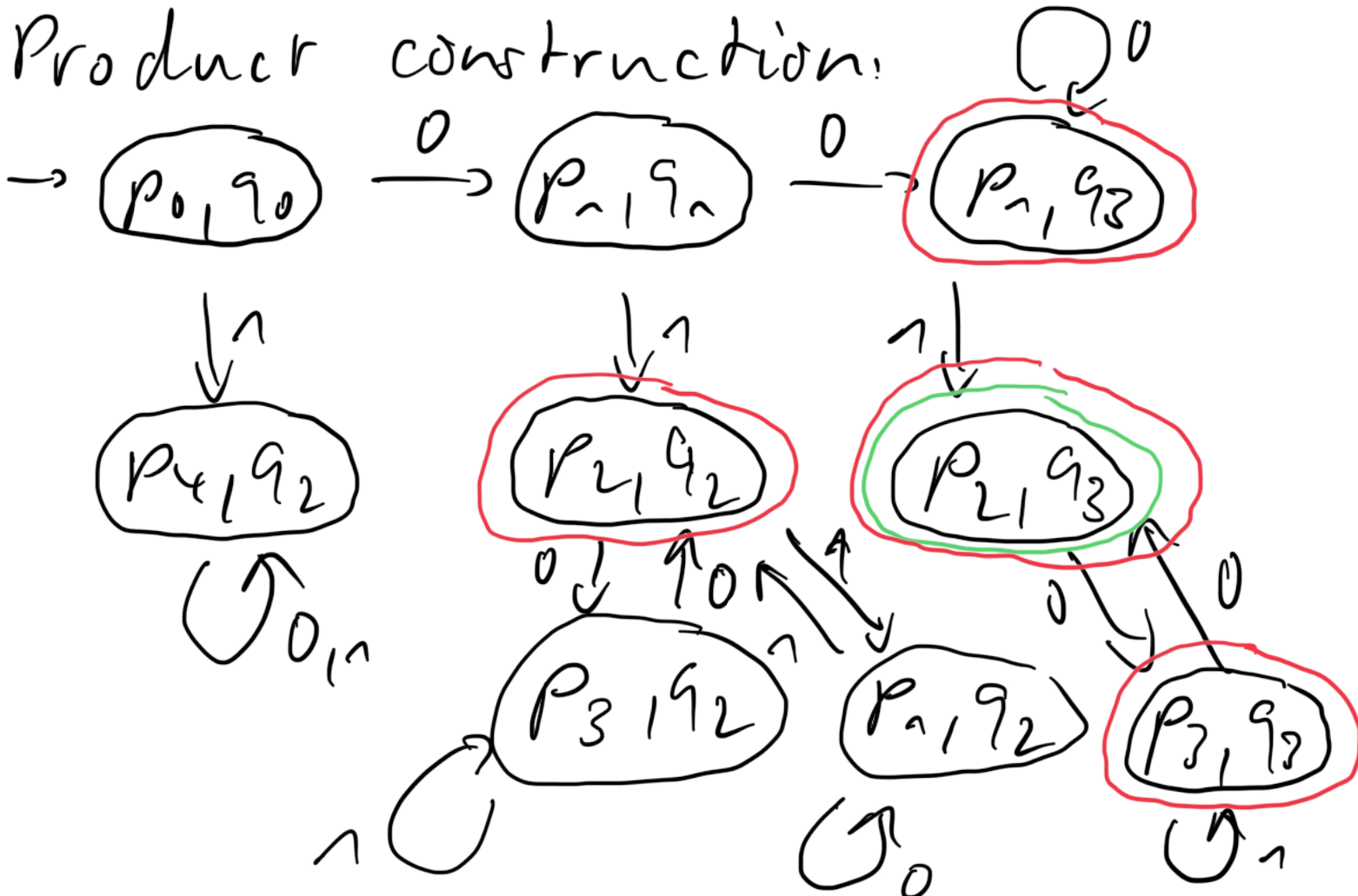
A3: Operations on Automata

Task: Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be the following automata over $\Sigma = \{0, 1\}$:



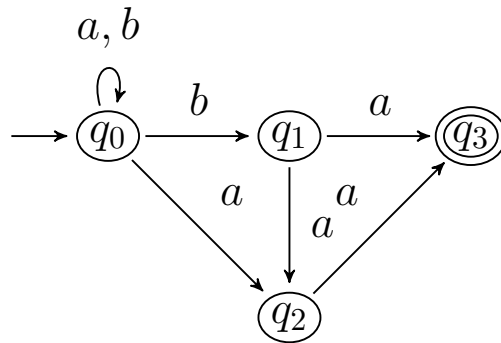
Construct two automata that respectively recognise the intersection and the union of the languages accepted by $\mathcal{A}_1$ and $\mathcal{A}_2$.

Product construction:



A4: Nondeterministic Finite Automata

Task: Let $\mathfrak{A}$ be the following NFA over $\Sigma := \{a, b\}$.



Determine the reachability sets $R_{\mathfrak{A}}(\varepsilon)$, $R_{\mathfrak{A}}(b)$, $R_{\mathfrak{A}}(ba)$, and $R_{\mathfrak{A}}(baa)$.

$$R_{\mathfrak{A}}(\varepsilon) = \{q_0\} \Rightarrow \varepsilon \notin L(\mathfrak{A})$$

$$R_{\mathfrak{A}}(b) = \{q_0, q_1\} \Rightarrow b \notin L(\mathfrak{A})$$

$$R_{\mathfrak{A}}(ba) = \{q_0, q_2, q_3\} \Rightarrow ba \in L(\mathfrak{A})$$

$$R_{\mathfrak{A}}(baa) = \{q_0, q_2, q_3\} \Rightarrow baa \in L(\mathfrak{A})$$

A5: Powerset Construction

Task: Apply the powerset construction to transform the following NFA $\mathcal{A}$ over $\Sigma := \{a, b, c\}$ into an equivalent DFA.

