

A7: Construction of Regular Expressions

Task: Give regular expressions that describe the following languages.

(a) $L := \{w \in \{a, b\}^* \mid |w| \text{ divisible by } 3\}$

(b) $L := \{w \in \{a, b, c\}^* \mid w \text{ does not contain } a, b, \text{ or } c\}$

(c) $L := \{w \in \{a, b\}^* \mid \text{substring } ab \text{ occurs exactly twice in } w, \text{ but not at the end}\}$

Solutions:

$$(a) ((a|b) \cdot (a|b) \cdot (a|b))^*$$

$$\alpha^+ := \alpha \alpha^*$$

$$(b) (b|c)^* | (a|c)^* | (a|b)^*$$

$$(c) b^* a^+ b^+ \quad a^+ b^+ (b^* a^+ | b^+ a^*)$$

A11: "Toolchain"

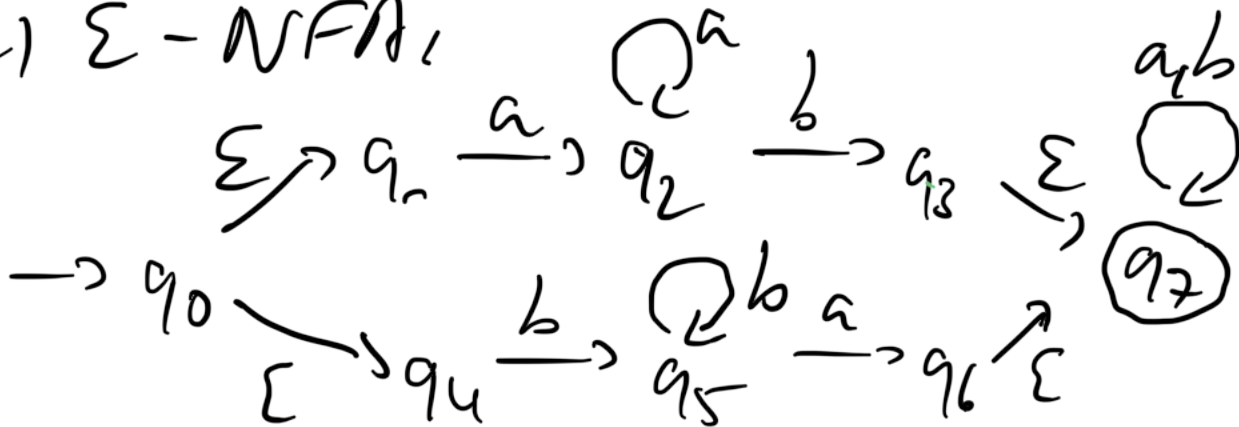
Task: Construct a DFA accepting the language

$$L = \{w \in \{a, b\}^* \mid w \text{ contains at least one } a \text{ and at least one } b\}.$$

(1) Reg. expression:

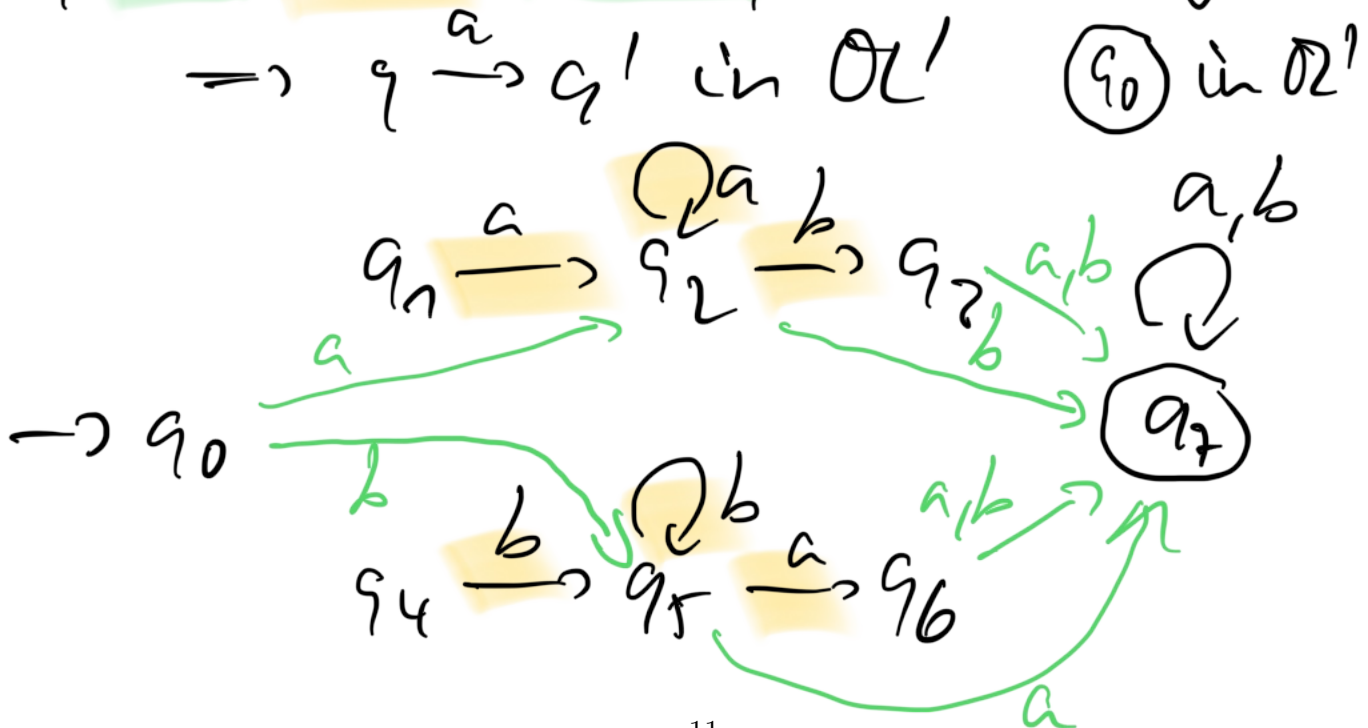
$$\alpha = (a^+b \mid b^+a) \cdot (a/b)^k$$

(2) Σ -NFA:



(3) Σ -elimination: $q_0 \xrightarrow{\Sigma} q_1$

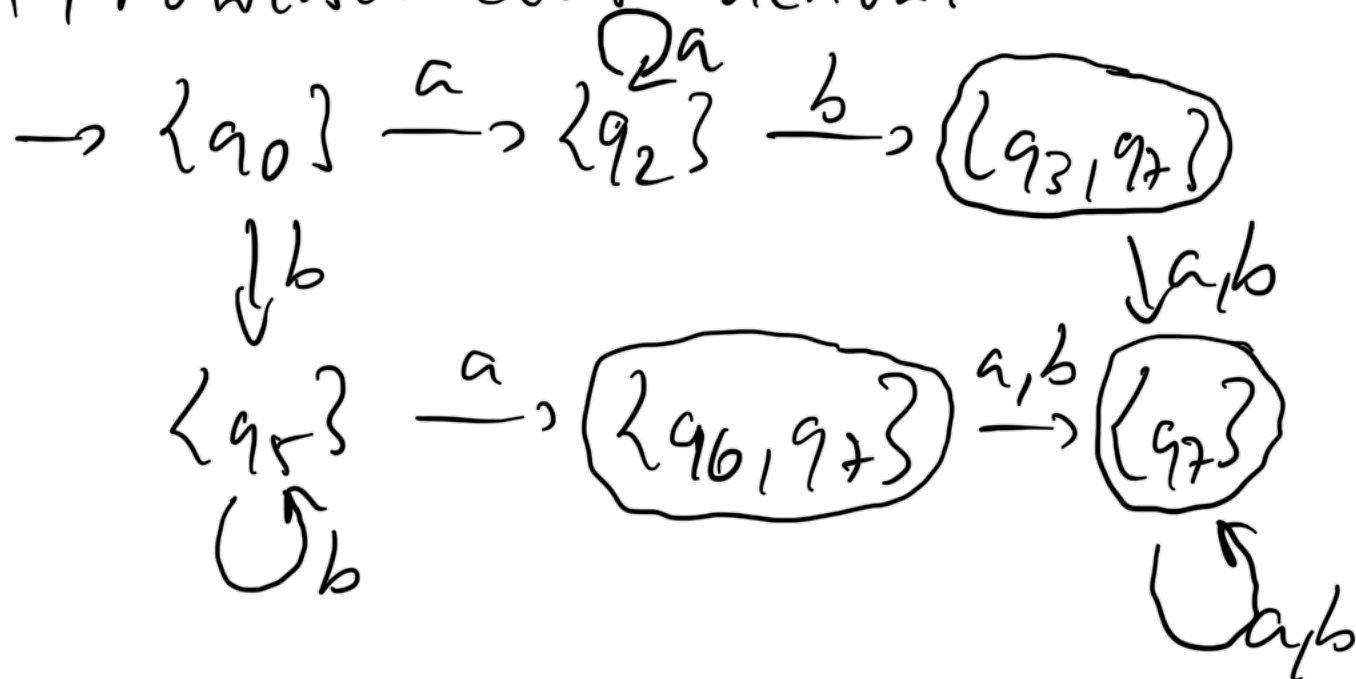
$$q \xrightarrow{\Sigma} q' \text{ in } \mathcal{A} \Rightarrow q \xrightarrow{a} q' \text{ in } \mathcal{A}' \quad (q_0 \text{ in } \mathcal{A}')$$



A12: "Toolchain"

Task: Construct a DFA accepting the language described by $\alpha = (ab^*)^*$.

(4) Powerset construction:



Observation: q_1, q_4 not reachable

(5) Minimization:

Merge q, q' with $q \sim q'$

where $q \sim q' \Leftrightarrow \forall w \in \Sigma^*$:

$$\delta^*(q, w) \in F \Leftrightarrow \delta^*(q', w) \in F$$

Thus: $q \not\sim q'$

$$\Leftrightarrow \exists w \in \Sigma^*, \delta^*(q, w) \in F \\ \text{but } \delta^*(q', w) \notin F$$

(or vice versa)

Matrix for equivalence check.

	q_0	q_2	q_5	(q_{37})	(q_{67})	(q_7)
q_0		① b	a	Σ	Σ	Σ
q_2			a	Σ	Σ	Σ
q_5				Σ	Σ	Σ
(q_{37})						
(q_{67})						
(q_7)						

① $q_0 \xrightarrow{b} q_5$, $q_2 \xrightarrow{b} q_{37}$

$q_5 \not\sim q_{37}$ (distinguished by Σ)

$\Rightarrow q_0 \not\sim q_2$ (distinguished by b)

Altogether, $q_{37} \sim q_{67} \sim q_7$

Minimized automaton:

